Math 2900 – Spring 2008
Homework VII
Due Mar 21

Problem 1: (a) Show that for \( n = 3 \) the general solution of wave equation with spherical symmetry about the origin has the form: \( u = r^{-1}(F(r + ct) + G(r - ct)) \), \( r = |x| \).
(b) Show that the solution with initial data \( u = 0, u_t = g(r) \), where \( g \) is even function of \( r \), is given by
\[
 u = \frac{1}{2cr} \int_{r-ct}^{r+ct} \rho g(\rho) d\rho 
\]
(\( \ast \))
(c) For
\[
g(r) = \begin{cases} 
1 & 0 < r < a \\
0 & r > a 
\end{cases}
\]
find \( u \) explicitly from (\( \ast \)) in every region of the \( xt \)-space. Show that \( u \) is discontinuous at \((0, a/c)\) (focusing discontinuity).

Problem 2: Consider the initial value problem for \( n = 5 \)
\[
u_{tt} - c^2 \Delta_x u = 0, \quad u(x,0) = f(x), \quad u_t(x,0) = g(x)
\]
(a) Show that \( N(x,r,t) = r^2 \frac{\partial}{\partial r} M_u(x,r,t) + 3rM_u(x,r,t) \) is a solution of \( N_{tt} - c^2 N_{rr} = 0 \) and find \( N \) from its initial data in terms of \( M_f \) and \( M_g \).
(b) Show that \( u(x,t) = \lim_{r \to 0} \frac{N(x,r,t)}{3r} = \left( \frac{t^2}{3} \frac{\partial}{\partial t} + \frac{\partial}{\partial t} \right) \frac{t^2}{3} \frac{\partial}{\partial t} M_f(x,ct) \)

Problem 3: Show that the solution of 2-dimensional IVP
\[
v_{tt} = c^2 (v_{x_1 x_3} + v_{x_2 x_3}) - \lambda^2 c^2 v, \quad v(x_1, x_2, 0) = \phi(x_1, x_2), \quad v_t(x_1, x_2, 0) = \psi(x_1, x_2)
\]
can be obtained from the solution of the 3-dimensional IVP
\[
u_{tt} - c^2 \Delta_x v = 0, \quad v(x,0) = f(x), \quad v_t(x,0) = g(x)
\]
by using the substitution
\[
u(x_1, x_2, x_3, t) = e^{i\lambda z} v(x_1, x_2, t).
\]
Write out the solution for \( v \).