Math 2900 – Spring 2008
Homework VII
Due Mar 21

Problem 1*:* (a) Show that for $n = 3$ the general solution of wave equation with spherical symmetry about the origin has the form: $u = r^{-1}\left(F(r+ct) + G(r-ct)\right)$, $r = |x|$.

(b) Show that the solution with initial data $u = 0, u_t = g(r)$, where *g* is even function of *r*, is given by

$$u = \frac{1}{2cr}\int_{r-ct}^{r+ct} \rho g(\rho) d\rho \qquad (*)$$

(c) For

$$g(r) = \begin{cases} 1 & 0 < r < a \\ 0 & r > a \end{cases}$$

find *u* explicitly from (*) in every region of the *xt*-space. Show that *u* is discontinuous at $(0, a/c)$ (focusing discontinuity).

Problem 2: Consider the initial value problem for $n = 5$

$$u_{tt} - c^2 \Delta_x u = 0, \quad u(x,0) = f(x), \quad u_t(x,0) = g(x)$$

(a) Show that $N(x,r,t) = r^2 \frac{\partial}{\partial r} M_u(x,r,t) + 3rM_u(x,r,t)$ is a solution of $N_{tt} - c^2 N_{rr} = 0$ and find *N* from its initial data in terms of M_f and M_g.

(b) Show that $u(x,t) = \lim_{r \to 0} \frac{N(x,r,t)}{3r} = \left(\frac{t^2}{3}\frac{\partial}{\partial t} + t\right) M_g(x,ct) + \frac{\partial}{\partial t}\left(\frac{t^2}{3}\frac{\partial}{\partial t} + t\right) M_f(x,ct)$

Problem 3: Show that the solution of 2-dimensional IVP

$$v_{tt} = c^2(v_{x_1x_1} + v_{x_2x_2}) - \lambda^2 c^2 v, \quad v(x_1,x_2,0) = \phi(x_1,x_2), \quad v_t(x_1,x_2,0) = \psi(x_1,x_2)$$

can be obtained from the solution of the 3-dimensional IVP

$$u_{tt} - c^2 \Delta_x u = 0, \quad u(x,0) = f(x), \quad u_t(x,0) = g(x)$$

by using the substitution

$$u(x_1,x_2,x_3,t) = e^{i\lambda x_3} v(x_1,x_2,t).$$

Write out the solution for *v*.