

Math 2900 – Spring 2008
Homework VI
Due Feb 22

Problem 1: Show that for a continuous function f the expression $f(x - ct)$ is a weak solution of the PDE $u_t + cu_x = 0$.

Problem 2: Show that a linear differential operator L on \mathbb{R}^n is translation invariant if and only if it has constant coefficients. (Consider the action of L on x^α .)

Problem 3: (Converse of the mean value theorem.) Suppose $u \in C^2(\Omega)$ and $x \in \Omega$. Show that

$$\Delta u(\xi) = \lim_{\rho \rightarrow 0} \frac{2n}{\rho^2} \left[\frac{1}{\omega_n \rho^{n-1}} \int_{S(\xi, \rho)} u(x) dS_x - u(\xi) \right]$$

Problem 4: Find all spherically symmetric solutions of the biharmonic equation $\Delta^2 u = 0$. Find fundamental solution(s) for $n \geq 2$.