Math 2900 – Spring 2008
Homework V
Due Feb 15

Problem 1: Show that the equation $xu_y - yu_x = x^2 + y^2$ has no continuous solutions in any neighborhood of (0,0). (Hint: convert to polar coordinates.)

Problem 2: Carry out explicitly the reduction of the Cauchy problem for the equation $u_t + u^T \nabla u = f$ in 2 spatial dimensions with $u = (u_1, u_2)$ to a first order system needed in the proof of Cauchy-Kowalevski theorem, assuming $u(x_1, x_2, 0) = g(x_1, x_2)$.

Problem 3: Show that the function

$$u(x_1, x_2) = \begin{cases} \frac{1}{2} & \text{for } |x_1 - \xi_1| < \xi_2 - x_2 \\ 0 & \text{otherwise} \end{cases}$$

Is a fundamental solution for $L = \frac{\partial^2}{\partial x_2^2} - \frac{\partial^2}{\partial x_1^2}$ with pole (ξ_1, ξ_2)

Problem 4: Verify:

$$\int_\Omega \left(v\Delta^2 u - u\Delta^2 v\right) dx = \int_\Omega \left(v\frac{d\Delta u}{dn} - \Delta u \frac{dv}{dn} - u\frac{d\Delta v}{dn} + \Delta v \frac{du}{dn} \right) dS$$