Math 2900 – Spring 2008
Homework III
Due Feb 1

Problem 1: Let $f(x)$ and $g(x)$ have compact support. Show that the solution $u(x,t)$ of

$$u_{tt} - c^2 u_{xx} = 0, \qquad u(x,0) = f(x), \quad u_t(x,0) = g(x)$$

has compact support in x for each fixed t. Show that the functions F, G in the decomposition $u(x,t) = F(x+ct) + G(x-ct)$ have compact support only if

$$\int_{-\infty}^{\infty} g(x)dx = 0.$$

Problem 2: Solve the initial-boundary-value problem

$$u_{tt} = u_{xx}, \qquad \text{for } 0 < x < \pi,\ 0 < t$$

$$u = 0, \qquad \text{for } x = 0, \pi;\ 0 < t$$

$$u = 1, \quad u_t = 0 \quad \text{for } 0 < \mathrm{x} < \pi;\ t = 0$$

Problem 3: (Legendre transformation) Let $u(x, y)$ be a solution of a quasi-linear equation

$$a(u_{x,}u_y)u_{xx} + 2b(u_{x,}u_y)u_{xy} + c(u_{x,}u_y)u_{yy} = 0$$

Introduce new independent variables ξ, η and a new function ϕ such that

$$\xi = u_x(x, y), \quad \eta = u_y(x, y), \quad \phi = xu_x + yu_y - u$$

Show that $\phi = \phi(\xi, \eta)$ satisfies $x = \phi_\xi$, $y = \phi_\eta$ and the linear differential equation

$$a(\xi,\eta)\phi_{\eta\eta} - 2b(\xi,\eta)\phi_{\xi\eta} + c(\xi,\eta)\phi_{\xi\xi} = 0$$