

Math 2900 – Spring 2008
Homework I
Due Jan 18

Problem 1: Solve the following initial value problems and verify solution

(a) $u_x + u_y = u^2$, $u(x,0) = \cos x$

(b) $u_y = xuu_x$, $u(x,0) = x$

Problem 2: Solve the Cauchy problem

$$xu_x - yu_y = 0, \quad x = y = z = s$$

Problem 3: Show that the solution of the following quasi-linear PDE with initial condition

$$u_y + a(u)u_x = 0, \quad u(x,0) = h(x)$$

is given implicitly by

$$u = h(x - a(u)y)$$

Find out under what condition the solution becomes singular at some positive y .

Problem 4: Show that the function $u(x, y)$ defined by

$$u = \begin{cases} -\frac{2}{3}(y + \sqrt{3x + y^2}) & \text{for } 4x + y^2 > 0, y \geq 0 \\ 0 & \text{for } 4x + y^2 < 0, y \geq 0 \end{cases}$$

is a weak solution of $\frac{\partial R(u)}{\partial y} + \frac{\partial S(u)}{\partial x} = 0$ for the choice $R(u) = u$, $S(u) = u^2 / 2$