Math 1080: Spring 2011

Homework #9 (due April 8)

Problem 1:

Calculate the Rayleigh quotients $r_1 = r(\mathbf{x}_1)$ and $r_2 = r(\mathbf{x}_2)$ for the following matrix **A** and vectors $\mathbf{x}_1$ and $\mathbf{x}_2$. How close are r_1 and r_2 to an eigenvalue of **A** ?

$$\mathbf{A} = \begin{bmatrix} 1 & 4 & 1 \\ 4 & 6 & 4 \\ 1 & 4 & 1 \end{bmatrix} \qquad \mathbf{x}_1 = \begin{bmatrix} 1.5 \\ 2 \\ 1 \end{bmatrix} \qquad \mathbf{x}_2 = \begin{bmatrix} 1 \\ 2.1 \\ 1 \end{bmatrix}$$

Problem 2:

Determine one eigenvalue of the following matrix using Rayleigh Quotient iteration, starting with initial guess $\mathbf{v}^{(0)} = [0 \quad 1]^{\mathrm{T}}$. Terminate iteration after 3 steps, i.e., after you obtain $\lambda^{(3)}$. What is the approximate eigenvector $\mathbf{v}^{(3)}$? What is the error of $\lambda^{(3)}$?

$$\mathbf{A} = \begin{bmatrix} 3 & -2 \\ -2 & 6 \end{bmatrix}$$

Problem 3:

Perform the first three steps of the QR algorithm (i.e., compute $\mathbf{A}^{(2)}$ and $\tilde{\mathbf{Q}}^{(2)}$) for the following matrix. How close are the diagonal elements of $\mathbf{A}^{(2)}$ to the eigenvalues of **A**?

$$\mathbf{A} = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix}$$

Computer Assignment 6:

a) Write a MATLAB function **`[lambda,v]=rayleigh(A,v0,mu,eps)`** that computes one eigenvalue of a square, symmetric matrix **A** using *Rayleigh Quotient iteration* with the initial guess **v0** and initial eigenvalue estimate **`mu`.** The program should terminate when the difference between two consecutive estimates, $|\lambda^{(k+1)} - \lambda^{(k)}|$, is smaller than the tolerance **`eps`**. The program should also print out $\lambda^{(k)}$ and $\mathbf{v}^{(k)}$ for each step of the iteration.

b) Calculate at least two different eigenvalues of the following matrix the **`rayleigh`** algorithm starting with two different initial guesses **`mu`** and using tolerance 10^{-6}. Then decrease the tolerance to 10^{-12} and note how many more steps the algorithm requires to converge. Compare this number with the number of steps needed for inverse iteration.

$$\mathbf{A} = \begin{bmatrix} 2 & 6 & 4 & -4 & -5 & -10 \\ 6 & 12 & -2 & 9 & 5 & 9 \\ 4 & -2 & 0 & -1 & -3 & 14 \\ -4 & 9 & -1 & 14 & -6 & 8 \\ -5 & 5 & -3 & -6 & 2 & 8 \\ -10 & 9 & 14 & 8 & 8 & 8 \end{bmatrix}$$