Math 1080: Spring 2011

Homework #8 (due April 1)

Problem 1:

Find the diagonalization $\mathbf{A} = \mathbf{X\Lambda X}^{-1}$ of the following matrix:

$$\mathbf{A} = \begin{bmatrix} -1 & 0 & -4 & 0 \\ 0 & 2 & -1 & 0 \\ 0 & 0 & 3 & 0 \\ 0 & 3 & 3 & -1 \end{bmatrix}$$

Problem 2:

Find the Schurr decomposition $\mathbf{A} = \mathbf{QTQ}^{\mathrm{T}}$ for the following <u>symmetric</u> matrix **A:**

$$\mathbf{A} = \begin{bmatrix} 4 & -2 & 1 \\ -2 & 4 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

Computer Assignment 5:

a) Write a MATLAB function **`[lambda,v]=poweriter(A,v0,eps)`** that computes one eigenvalue of a square, symmetric matrix **A** using *power iteration* and Rayleigh quotient with the initial guess **v0**. The program should terminate when the difference between two consecutive estimates, $\left|\lambda^{(k+1)} - \lambda^{(k)}\right|$, is smaller than the tolerance **`eps`**. The program should also print out $\lambda^{(k)}$ and $\mathbf{v}^{(k)}$ for each step of the iteration.

b) Write a MATLAB function **`[lambda,v]=inverseiter(A,v0,mu,eps)`** that computes one eigenvalue of a square, symmetric matrix **A** using *inverse iteration* and Rayleigh quotient with the initial guess **v0** and **`mu`**. The program should terminate when $\left|\lambda^{(k+1)} - \lambda^{(k)}\right|$ is smaller than **`eps`** and it should print out $\lambda^{(k)}$ and $\mathbf{v}^{(k)}$ for each step of the iteration.

c) Calculate at the leading eigenvalue of the following matrix using both algorithms starting with two different initial guesses **v0** using tolerance 10^{-6}. With inverse iteration find at least two different eigenvalues using different **`mu`**. Note how many iterations were needed in each case.

$$\mathbf{A} = \begin{bmatrix} 2 & 6 & 4 & -4 & -5 & -10 \\ 6 & 12 & -2 & 9 & 5 & 9 \\ 4 & -2 & 0 & -1 & -3 & 14 \\ -4 & 9 & -1 & 14 & -6 & 8 \\ -5 & 5 & -3 & -6 & 2 & 8 \\ -10 & 9 & 14 & 8 & 8 & 8 \end{bmatrix}$$