Math 1080: Spring 2011

Homework #7 (due March 16)

Problem 1:

Show that if $\mathbf{A} = \begin{bmatrix} a_{11} & \mathbf{w}^{\mathrm{T}} \\ \mathbf{w} & \mathbf{K} \end{bmatrix}$ is symmetric and positive definite, then $a_{11} > 0$ and both $\mathbf{K}$ and $\mathbf{K} - \mathbf{w}\mathbf{w}^{\mathrm{T}} / a_{11}$ are symmetric and positive definite.

(Hint: Use the definition of positive definite matrix. Assume $\mathbf{x} = \begin{bmatrix} x_1 \\ \mathbf{y} \end{bmatrix}$. Make an appropriate choice of x_1.)

Problem 2:

Solve the following system of equations by Cholesky factorization (if the coefficient matrix is positive definite) or by Gaussian elimination (otherwise)

$$\begin{aligned} 4x_1 + 2x_2 \qquad\quad - 2x_4 &= 6 \\ 2x_1 + 10x_2 - 6x_3 + 2x_4 &= 36 \\ -6x_2 + 8x_3 \qquad &= -30 \\ -2x_1 + 2x_2 \qquad\quad + 4x_4 &= 6 \end{aligned}$$

Computer Assignment 5:

a) Write a MATLAB function **`[R]=cholesky(A)`** that computes the Cholesky factorization of a square $m \times m$ matrix **A**. The output variable is a lower triangular $m \times m$ matrix **R**.

b) For the following matrix

$$\mathbf{A} = \begin{bmatrix} 10 & 3 & -1 & -3 & -9 \\ 3 & 6 & -5 & -3 & -1 \\ -1 & -5 & 9 & 2 & -1 \\ -3 & -3 & 2 & 2 & 2 \\ -9 & -1 & -1 & 2 & 9 \end{bmatrix}$$

compute two LU factorizations: (1) using Gaussian elimination with partial pivoting **`gausspivot`** and (2) Using the Cholesky factorization. Compute the relative accuracy of each method and compare them :

`Delta1 = norm(L*U-P*A)/norm(A);` in case (1)

`Delta2 = norm(R'*R- A)/norm(A);` in case (2)