Math 1080: Spring 2016
Homework #5
due Monday, February 29

Problem 1: Compute LU factorization of the matrix

\[
A = \begin{bmatrix}
1 & 2 & -1 & 4 \\
-2 & -2 & -1 & -4 \\
0 & -2 & -2 & 5 \\
-2 & 2 & -6 & 7
\end{bmatrix}
\]

Problem 2:
Solve the following system of equations by both LU factorization and QR factorization:

\[
\begin{align*}
2x_1 + 8x_2 + 3x_3 &= 2 \\
x_1 + 3x_2 + 2x_3 &= 5 \\
2x_1 + 7x_2 + 4x_3 &= 8 
\end{align*}
\]

Problem 3:
Compute the LU factorization with partial pivoting, (i.e., find \( P, L, U \) such that \( PA = LU \)) for the following matrix

\[
A = \begin{bmatrix}
1 & 1 & 1 \\
3 & -2 & -7 \\
3 & 2 & -1
\end{bmatrix}
\]

Computer Assignment 3:

a) Write a MATLAB function \([L,U]=\text{gauss}(A)\) that computes the LU factorization of a square \( m \times m \) matrix \( A \) using Gaussian elimination. The output variables are a lower triangular \( m \times m \) matrix \( L \) and an upper triangular \( m \times m \) matrix \( U \).

b) Write a MATLAB function \([L,U,P]=\text{gausspivot}(A)\) that computes the LU factorization of a square \( m \times m \) matrix \( A \) using Gaussian elimination with partial pivoting. The output variables are a lower triangular \( m \times m \) matrix \( L \), an upper triangular \( m \times m \) matrix \( U \), and an \( m \times m \) permutation matrix \( P \).
c) For the following matrix

\[
A = \begin{bmatrix}
-8 & -9 & 7 & 19 & 3 \\
9 & 10 & 9 & 1 & -16 \\
7 & 2 & -8 & -2 & 3 \\
19 & 8 & -18 & -8 & -3 \\
15 & 10 & 16 & -16 & -18 \\
\end{bmatrix}
\]

compute two LU factorizations: (1) using Gaussian elimination without pivoting \texttt{gauss}, and (2) using Gaussian elimination with partial pivoting \texttt{gausspivot}. Compute the relative accuracy of each method and compare them:

\[
\text{Delta}_1 = \frac{\text{norm}(L*U-A)}{\text{norm}(A)}; \quad \text{in case (1)}
\]

\[
\text{Delta}_2 = \frac{\text{norm}(L*U-P*A)}{\text{norm}(A)}; \quad \text{in case (2)}
\]