

Math 1080: Spring 2011
Homework #3 (due January 31)

Problem 1:

Show that the Householder reflector $\mathbf{F} = \mathbf{I} - 2\mathbf{w}\mathbf{w}^T$, with $\|\mathbf{w}\| = 1$, is symmetric and orthogonal. Find the eigenvalues and eigenvectors of \mathbf{F} .

Problem 2:

Use Householder orthogonalization procedure to find the QR factorization of

$$A = \begin{bmatrix} 1 & 4 & 1 \\ 0 & 2 & 1 \\ 1 & 0 & -1 \end{bmatrix}$$

Problem 3:

Find the *reduced* QR factorization of the matrix

$$A = \begin{bmatrix} 2 & 2 \\ -2 & 3 \\ 0 & 1 \\ 1 & 1 \end{bmatrix}$$

Computer Assignment 2:

- a) Write a MATLAB function `[Q,R]=house(A)` that computes full QR factorization of an $m \times n$ matrix \mathbf{A} with $m \geq n$ using Householder triangularization. The output variables are the orthogonal $m \times m$ matrix \mathbf{Q} , and the upper triangular $m \times n$ matrix \mathbf{R} .
- b) For the following matrix

$$\mathbf{Z} = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 7 \\ 4 & 2 & 3 \\ 4 & 2 & 2 \end{bmatrix}$$

compute three QR factorizations using (1) the Gram-Schmidt algorithm `gs`, (2) the Householder subroutine `house`, and (3) the built-in command `[Q,R]=qr(A)`.

Compare the results and comment on any differences you find.