

Math 1080: Spring 2011

Homework #2

Due Jan 21

Problem 1:

Let \mathbf{A} be an 3×3 matrix and let $\mathbf{A} = \hat{\mathbf{Q}}\hat{\mathbf{R}}$ be a reduced QR factorization. Show that \mathbf{A} has rank 3 (i.e., is non-singular) if and only if all the diagonal entries of $\hat{\mathbf{R}}$ are nonzero.

Problem 2:

Using Gram-Schmidt orthogonalization compute the QR factorization of the following matrix

$$\mathbf{A} = \begin{bmatrix} 12 & -20 & 41 \\ 9 & -15 & -63 \\ 20 & 50 & 35 \end{bmatrix}$$

Computer Assignment 1:

(a) Write a function $[\mathbf{Q}, \mathbf{R}] = \mathbf{gs}(\mathbf{A})$ that computes a reduced QR factorization $\mathbf{A} = \hat{\mathbf{Q}}\hat{\mathbf{R}}$ of an $m \times n$ matrix \mathbf{A} with $m \geq n$ using the Gram-Schmidt orthogonalization. The output variables are the $m \times n$ matrix \mathbf{Q} and $n \times n$ upper triangular matrix \mathbf{R} .

(b) Use the function to calculate the QR factorization of the following matrix

$$\mathbf{A} = \begin{bmatrix} 1 & 1 + \varepsilon & 1 + 2\varepsilon \\ 1 & 1 & 1 + \varepsilon \\ 1 & 1 & 1 \end{bmatrix}$$

with $\varepsilon = 10^{-10}, 10^{-12}, 10^{-14}$.

(c) Test the loss of orthogonality by calculating $\mathbf{E} = \mathbf{Q}^T \mathbf{Q} - \mathbf{I}$.

Hand in the printout of your program and the resulting \mathbf{Q} and \mathbf{E} matrices for each ε .

Hints for writing the program in MATLAB:

1. Create a new m-file called `gs.m`
2. The first line of the file contains the heading: `function [Q,R] = gs(A)`
3. The next command finds out the size of the matrix \mathbf{A} : `[m,n] = size(A);`
4. Now put in the lines of the algorithm for modified Gram-Schmidt. Remember that the j -th column of matrix \mathbf{A} is in MATLAB written as `A(:, j)`, the i -th row of matrix \mathbf{R} is written as `R(i, :)`.