Math 1080: Spring 2010
Homework #10 (due April 15)

Problem 1:

Let **Q** and **R** be the QR factors of a symmetric tridiagonal matrix **H**. Show that the product $\mathbf{K} = \mathbf{RQ}$ is again a symmetric tridiagonal matrix.

(Hint: Prove the symmetry of **K**. Show that **Q** has Hessenberg form and that the product of an upper triangular matrix and a Hessenberg matrix is again a Hessenberg matrix. Then use the symmetry of **K**.)

Problem 2:
Reduce the following matrix to Hessenberg form.

$$\mathbf{A} = \begin{bmatrix} 3 & 7 & -2 & -8 \\ -2 & 1 & 1 & 4 \\ 4 & 9 & 0 & 0 \\ 4 & -4 & -4 & 2 \end{bmatrix}$$

Computer Assignment 7:

a) Write a MATLAB function **`[Q,L]=qralgshift(A,eps)`** that computes the eigenvalues and eigenvectors of a square, symmetric $m \times m$ matrix **A** using QR algorithm with shift. The output variables are the $m \times m$ orthogonal matrix **Q** which columns are the eigenvectors of **A** and $m \times m$ matrix **L** that has the corresponding eigenvalues of **A** on the main diagonal. The program should terminate iteration when the norm of the offdiagonal elements of $\mathbf{A}^{(k)}$, i.e., `norm(A-diag(diag(A))),` is smaller than 10^{-6}.

b) Write a MATLAB function **`[Q,H]=hessenberg(A)`** that computes the Hessenberg form of $m \times m$ matrix **A**. The output variables are $m \times m$ orthogonal matrix **Q** and $m \times m$ matrix **H** similar to **A** that is in Hessenberg form.

c) Calculate the eigenvalues and eigenvectors of using (1) **`qralgshift`** algorithm only and (2) a combination of **`hessenberg`** and **`qralgshift`**.

$$\mathbf{A} = \begin{bmatrix} 3 & -2 & 12 & -9 & -6 & -9 \\ -2 & 3 & 1 & 8 & -1 & -1 \\ 12 & 1 & -9 & -3 & -3 & -11 \\ -9 & 8 & -3 & 4 & 4 & 17 \\ -6 & -1 & -3 & 4 & 15 & -3 \\ -9 & -1 & -11 & 17 & -3 & 7 \end{bmatrix}$$

Record the number of iterations needed to achieve the desired accuracy. Compare the two methods.