

**Math 1080: Spring 2011**

**Homework #1**

**Due Jan 14**

*Problem 1:*

Consider the matrix

$$\mathbf{Q} = \begin{bmatrix} 1/2 & 0 & \sqrt{3}/2 \\ 0 & 1 & 0 \\ -\sqrt{3}/2 & 0 & 1/2 \end{bmatrix}$$

Show that  $\mathbf{Q}$  is orthogonal. What transformation of  $\mathbb{R}^3$  does it correspond to ?

(Hint: Find the vector  $\mathbf{a}$  that is invariant under  $\mathbf{Q}$ . Pick a vector  $\mathbf{b}$  orthogonal to  $\mathbf{a}$ . Find the angle  $\alpha$  between  $\mathbf{b}$  and  $\mathbf{Q}\mathbf{b}$ . If this angle is independent of the choice of  $\mathbf{b}$ , then  $\mathbf{Q}$  corresponds to a rotation about  $\mathbf{a}$  by the angle  $\alpha$ . Think about other possibilities.)

*Problem 2:*

Find the  $3 \times 3$  orthogonal matrix that corresponds to the reflection of  $\mathbb{R}^3$  through the plane  $x + y = 0$

*Problem 3:*

Find the orthogonal projector  $\mathbf{P}$  onto  $\text{range}(\mathbf{A})$  where

$$\mathbf{A} = \begin{bmatrix} 1 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}$$

What is the nullspace of  $\mathbf{P}$ ? What is the image under  $\mathbf{P}$  of the vector  $[1 \ 2 \ 4]^T$  ?