Continuum Shell Models for Two-Phase Lipid Bi-layer Membrane Vesicles

Timothy J. Healey
Department of Theoretical & Applied Mechanics
Cornell University
Ithaca, NY 14853, USA
tjh10@cornell.edu

In this talk we present and analyze a new model for two-phase lipid bi-layer membranes. The work is motivated by the recent experiments of Baumgart, et. al. [1], on liposome vesicles – in this case, giant unilamellar vesicles (GUVs) - demonstrating remarkable two-phase patterns under the variation of osmotic pressure and/or temperature. The two phases observed in [1] are each fluid-like - an ordered phase and a disordered phase. In particular, the membrane is observed to be slightly thicker in the ordered phase.

Heretofore in the biophysics literature, a Ginzburg-Landau type model has been considered, with an ad-hoc term that engenders intrinsic curvature when the phase field variable is non-zero. Here we propose a rational model for pressure-induced phase transitions, incorporating the associated change in thickness within the bending potential energy. Our model incorporates no intrinsic curvature in either phase.

We derive the equilibrium equations and consider a nominally spherical vesicle under *inflationary* pressure $p > 0$ (so that we do not confuse things with standard shell buckling under compression). We then linearize the equations about the spherically symmetrical state, looking for the existence of non-spherical states. We perform a rigorous nonlinear bifurcation analysis, using well-known group-theoretic strategies, showing the existence of bifurcated equilibria. The latter, representing phase-nucleated states from the homogeneous vesicle, bear striking similarity to many of the non-spherical states observed in the experiments of [1]. We further present some numerical results for some axisymmetric two-phase configurations.

[1] T. Baumgart, S. T. Hess, and W. Webb, “Imaging Coexisting Fluid Domains in Biomembrane Models Coupling Curvature and Line Tension”, *Nature*, 425. (2003) p. 821.