Plasticity-like Models for Growth

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Several proposed models for growth have been based on analogies to plasticity theory. For example, the idea of a (possibly incompatible) stress-free intermediate configuration that decomposes the total deformation into a growth part and an elastic part corresponds to the classical elastic-plastic decomposition in plasticity. This procedure also requires constitutive growth laws analogous to plastic flow rules. Further, constitutive relations have been formulated relative to the stress-free intermediate configuration, which serves as a "natural reference state" as in plasticity. Epstein and Maugin (2000) explicitly acknowledged the analogy of growth to plasticity and discussed the role of the configuration or Eshelby stress.

The analogy to plasticity appears to be quite fruitful. Here we further extend that analogy and use recent ideas from plasticity to develop a mechanical theory of volumetric growth of a single, bulk material. Importantly, recent theories of crystalline solids postulate additional forces or couples subject to supplemental balance laws. These additional generalized forces can be viewed as acting in the intermediate configuration where they drive plastic flow. Here, we employ that viewpoint and postulate two distinct sets of forces subject to their own balance laws:

1) Standard forces that act in the current configuration and drive mechanical deformation; and

2) Growth couples that act in the intermediate configuration and drive growth.

This growth couple balance replaces the typically assumed flow rule.

Another common idea in plasticity is that incompatibilities lead to higher-order stresses. Since both the growth and elastic contributions to the total deformation gradient are typically incompatible---that is, not gradients---we account for couple stresses when formulating the couple balance. The growth model of DiCarlo & Quiligotti (2002) is a special case of this model with negligible couple stresses, corresponding to the plasticity model of Cermelli, Fried and Sellers (2001).

Constitutive relations are postulated for the proposed model. Numerical simulations illustrate the possible response.