Comment on problems on page 29.

These are trickier than they seem. The goal is to justify each step by reference to one of the axioms or theorems, or the assumptions in the problem. Here are a couple of examples. These might not be the shortest or best solutions – can you do better?

Solution to # 3(a), pg. 29., which says: Solve the equation $2x + 5 = 8$.

Suppose that $2x + 5 = 8$. Then

$$\begin{aligned}
x &= 1 \cdot x \text{ (by M3)} \\
&= \left(\frac{1}{2} \cdot 2\right) x \text{ by M4} \\
&= \frac{1}{2}(2x) \text{ by M2} \\
&= \frac{1}{2}(2x + 0) \text{ by A3} \\
&= \frac{1}{2}(2x + (5 + (-5))) \text{ by A4} \\
&= \frac{1}{2}((2x + 5) + (-5)) \text{ by A2} \\
&= \frac{1}{2}(8 + (-5)) \text{ since } 2x + 5 = 8 \\
&= \frac{1}{2} \cdot 3 = \frac{3}{2} \text{ by basic arithmetic, which we assume.}
\end{aligned}$$

In the future (after this section, unless instructed otherwise on an exam), you will be able to solve this equation by ordinary algebra:

$$\begin{aligned}
2x + 5 &= 8 \\
2x &= 3 \text{ (subtract 5 from each side)} \\
x &= \frac{3}{2} \text{ (divide each side by 2)}
\end{aligned}$$

However, as the earlier proof shows, each of these steps uses several of the axioms. Our goal here is to break it down so that only one assumption is used at each step. For example, none of the axioms states explicitly that we can subtract a number from each side of an equation.

2nd example: pg. 29, # 3d. Solve $(x-1)(x+2)=0$.

From Theorem 2.1.3(b), we see that either $x-1=0$ or $x+2=0$. If $x-1=0$, then

$$\begin{aligned} x &= x+0 \text{ by A3} \\ &= x+(1+(-1)) \text{ by A4} \\ &= x+((-1)+1) \text{ by A1} \\ &= (x+(-1))+1 \text{ by A2} \\ &= (x-1)+1 \text{ by the notation introduced on page 24} \\ &= 0+1 \text{ since we are assuming that } x-1=0 \\ &= 1 \text{ by A3.} \end{aligned}$$

A similar argument shows that if $x+2=0$ then $x=-2$. (As I have, you could omit the details here if this were an assigned problem.)