Math 0450
Honors intro to analysis
Spring, 2009
Notes 18

(Revised homework assignment)

1 Integrals

Read: Professor Beatrous's notes,"integrals.pdf" on the website The presentation in the text is very much more complicated, and I believe that its disadvantages outweigh its advantages.

Perhaps the most complicated part of BN (Beatrous's notes) is the notation, but it's a lot simpler than that in the text. It takes awhile to get familiar with $\mathcal{P}$, $L(f,\mathcal{P})$, and $U(f,\mathcal{P})$. Also, the notation $L\int_a^b f$ and $U\int_a^b f$ may seem a bit strange.

I'm not sure how standard this second notation is. Alternatives (for the lower sum) might be

$$L\left(\int_a^b f\right), \text{ or simply } Lf \text{ or } L(f).$$

But the former takes more space, and the latter doesn't show any connection to integrals. Some authors use

$$L\left(\sum f\right),$$

which I think might be my preference here. But in more advanced math it is quite common to leave out parentheses in cases like these. For example, the operator notation for derivatives, which is Df (which equals f') is more common than $D(f)$.

Another notation which may be unfamiliar is to use $\int_a^b f$, instead of $\int_a^b f(x)\,dx$. The latter is supposed to be reminiscent of an expression like

$$\sum_{j=1}^{n} f\left(x_j'\right)\Delta_i x,$$

with $\Delta_i x$ stands for $x_j - x_{j-1}$. If the partitioning intervals are all of the same length, then often

$$\sum_{j=1}^{n} f\left(x_j'\right)\Delta x$$

is used. Here the x_j' are points in $[x_{j-1}, x_j]$.

The main value of the $\int_a^b f(x)\,dx$ notation is when the function f is given by a formula like

$$f(x) = cx,$$

where there is more than one parameter. We could have $\int_a^b x^2 y\ dx$ and $\int_a^b x^2 y\ dy$, and these are different. (Check that.) But usually mathematicians like as simple notation as possible, so that when there is no ambiguity, $\int_a^b f$ is used.

Often one thinks of an integral as a "limit of sums". This is seen in BN, for example, when $\int_0^1 x$ is calculated. This is helpful when thinking about how to prove such rules as: $\int_a^b f + \int_b^c f = \int_a^c f$, or "If $f \leq g$ on $[a, b]$, then $\int_a^b f \leq \int_a^b g$."

The best way to become familiar with the notation is to do problems using it, so here are some. (There are also some problems from chapter 6.)

2 Homework, Due April 9. (Revised assignment, combining two weeks.)

#1. Let $f(x) = x^2$ for $0 \leq x \leq 1$. Find $\int_0^1 f$ using the definition of integral in BN.

#2 Suppose that $f : [0, 1] \to R$ is defined by

$$f(x) = 1 \text{ if } x = \frac{1}{2}$$
$$0 \text{ otherwise.}$$

Prove that $\int_0^1 f = 0$.

pg. 167, #8(a,c)

pg. 175 , #3b (include a graph), #6

pg. 209, #8 (state how this is related to problem # 2 above), #10, 12 ($\mathcal{R}[0, 1]$ denotes the set of functions which are integrable on $[0, 1]$.), #18

pg. 217, # 9(c), 11(b), 18(b) (uses problem 17; see also Theorem 7.3.8)