Complex Variables: Quiz 3 3/18/10

Name:

Question 1

Calculate the following limits:

- \( \lim_{z \to 1} \left( \frac{z^3 - 1}{\sin(z - 1) \cos(z - 1)} \right) \)
- \( \lim_{z \to 0} (1 + z \sin(z))^{\frac{1}{z^2}} \)

Question 2

Let \( \beta = \frac{dz}{z + 1} \).
Find the contour integral of \( \beta \) taken around the following contours all traced once around counter-clockwise:

- \( A \): the circle \( |z + 1| = 2 \)
- \( B \): the circle \( |z| = \frac{1}{2} \)
- \( C \): the square \( PQRS \) with vertices \( P = (2, 2), Q = (-2, 2), R = (-2, -2), S = (2, -2) \).

Question 3

Let \( \alpha = (z^3 + 2iz - 2iz \, zdz) \).
Find the integral of \( \alpha \) from \( A = (0, 0) \) to \( B = (2, 4) \) along two contours:

- Along the parabola \( y = x^2 \).
- Vertically from \( A \) to the point \( C = (0, 4) \) and then horizontally to \( B \).

Explain the difference between these two results, using Green’s Theorem.