Theoretical Mathematics II, Quiz 1, 9/10/9

Name:

Question 1
Let \( f(x) = \sqrt{x} \).
Prove from first principles the limit: \( \lim_{x \to 1} f(x) = 2 \).
Also find an open interval \( \mathbb{J} \), containing the number 4, such that, for any \( x \in \mathbb{J} \), \( f(x) \) is within 0.1 of 2.

Question 2
Prove from first principles the following limit:
\[
\lim_{x \to 1} \left( \frac{x}{x + 4} \right) = \frac{1}{5}.
\]

Question 3
Determine, with proof, each of the following limits, or prove that the limit in question does not exist:

- \( A = \lim_{x \to 2} \left( \frac{x^2 - 6x + 8}{x^2 - 4} \right) \)

- \( B = \lim_{x \to 0} \cos \left( \frac{1}{x} \right) \)

Question 4
Let \( h : \mathbb{R} \to \mathbb{R} \) be such that \( h(x) = x \) if \( x \) is rational.
Suppose that, for any real \( c \), the limit \( \lim_{x \to c} h(x) \) exists and equals \( h(c) \).
Prove that \( h(x) = x \) for all real numbers \( x \).