Complex Variables, Summer 2009
Homework Assignments

Homework 1, due Thursday June 25th

Prepare for the quiz on Thursday: material from chapter one of Spiegel: the geometry of complex numbers.
Do the following four problems and the four problems from chapter one of Spiegel: 71, 73, 74, 123 (omit the fourth part and the last two parts of this problem).

Speigel 1.71

Describe and graph the following curves in the complex plane:

- $|z - i| = 2$
- $|z + 2i| + |z - 2i| = 6$
- $|z - 3| - |z + 3| = 4$
- $z(\overline{z} + 2) = 3$
- $\Im(z^2) = 4$

Speigel 1.73

Sketch the following regions in the complex plane and for each region discuss whether or not it is closed, open, bounded, unbounded, compact or non-compact.

- $1 < |z + i| \leq 2$
- $\Re(z^2) > 1$
- $|z + 3i| > 4$
- $|z + 2 - 3i| + |z - 2 + 3i| < 10$

Spiegel 1.74

Show that the Cartesian equation of the ellipse $|z + 3| + |z - 3| = 10$ can be written:

$$\frac{x^2}{25} + \frac{y^2}{16} = 1.$$

Speigel 1.123

Let regions $\mathbb{A}$, $\mathbb{B}$ and $\mathbb{C}$ in the complex plane be given as follows:

$$\mathbb{A} = \{z \in \mathbb{C} : |z + i| < 3\} \quad \mathbb{B} = \{z \in \mathbb{C} : |z| < 5\} \quad \mathbb{C} = \{z \in \mathbb{C} : |z + 1| < 4\}$$

Sketch $\mathbb{A}$, $\mathbb{B}$ and $\mathbb{C}$ and the following sets:

- $\mathbb{A} \cap \mathbb{B} \cap \mathbb{C}$
- $\mathbb{A} \cup \mathbb{B} \cup \mathbb{C}$
- $\mathbb{A} \cap (\mathbb{B} \cup \mathbb{C})$
- $(\mathbb{A} \cup \mathbb{B}) \cap (\mathbb{B} \cup \mathbb{C})$

Question 1

Let $a = 3 + 2i$ and $b = 4 + 3i$.
Sketch the complex numbers a, b, $a + b$, $a - b$, a^2, ab, $a\overline{b}$, b^2, $\frac{a}{b}$ and $\frac{b}{a}$.
For each of these numbers, determine their modulus and argument.

Question 2

Find the solutions of the equation $z^5 = -32$ and all solutions of the equation $z^{10} = 1024$ and plot the solutions in the complex plane.

Question 3

A complex number z obeys the relations $|z + i| = 3\sqrt{2}$ and $|z - 3| = 4$.
What can we say about the number z?
In particular is z unique? Explain your answer graphically.

Question 4

Solve (algebraically) the equations $z^2 = -3 + 4i$ and $z^2 = -3 - 4i$ and plot the solutions on the complex plane.

Homework 2, due Monday July 6th

Prepare for the quiz on Thursday: material from chapters one and two of Spiegel: the geometry of complex numbers and complex functions.
Do the following three problems from chapter one of Spiegel: 130, 161, 167.
Also do the following five problems from chapter two of Spiegel: 50, 51, 53, 56, 65.
Also for problem 53, show that the functions u and v of each part of the question are harmonic, so are solutions of Laplace's equation.

Homework 3, due Monday July 13th

Prepare for the exam on Thursday 9th July: material from chapters one to three inclusive of Spiegel: the geometry of complex numbers and complex functions, complex derivatives and the Cauchy-Riemann equations.

Also do the following eight problems from chapter three of Spiegel:
53, 54, 78, 79, 104, 112, 115, 117.

Homework 4, due Monday July 20th

Prepare for the quiz on Thursday 16th July: complex integration and Cauchy-Riemann equations.

Also do the following eight problems from chapter four of Spiegel:
33, 39, 43, 49, 50, 72, 78, 92.

Homeworks 5/6, due Wednesday July 29th

Prepare for the quiz on Thursday 23rd July and the final Thursday 30th July: series and contour integration and conformal transformations.

Also do the following problems:
From chapter five of Spiegel: 56, 62, 80.
From chapter six of Spiegel: 79, 92, 96.
From chapter seven of Spiegel: 76, 95, 100.
From chapter eight of Spiegel: 32, 34, 35.