Differential Equations, Exam 2, 3/30/7

Name:

Show your work
25 points per question; the best four questions will count

Question 1

Classify the phase portraits (sink, source, spiral, node, saddle, etc.) for the system $X' = AX$, for the following cases and for each discuss whether or not its type is generic:

- $A = \begin{bmatrix} -5 & 3 \\ 5 & -7 \end{bmatrix}$

- $A = \begin{bmatrix} 3 & 2 \\ -5 & -3 \end{bmatrix}$
Question 2

Solve the following differential system and discuss the behavior of the solution, with a sketch.
Also discuss whether or not the system is generic;

\[ X' = AX, \quad A = \begin{pmatrix} 3 & -2 \\ -2 & 0 \end{pmatrix}, \quad X(0) = \begin{pmatrix} 5 \\ 5 \end{pmatrix} \]
Question 3

Tanks A and B each contain two hundred liters of brine.
Two liters per minute of pure water flows into tank A.
Two liters per minute of pure water flows into tank B.
Five liters per minute of well-mixed fluid flows out from tank A along a pipe
into tank B.
The well-mixed fluid in tank B flows along a second pipe back into tank A
at a rate of three liters per minute.
Also four liters per minute of the well-mixed fluid empties from tank B into
a reservoir.
Initially tank A contains two kilograms of salt.
Initially tank B contains five kilograms of salt.
Determine the amount of salt in each tank as a function of time and discuss
your results.
If the reservoir is initially empty, also determine the amount of salt in the
reservoir as a function of time and discuss your results.
Question 4

Consider the following differential system:

\[ x' = x(3 - x - 2y), \quad y' = y(5 - x). \]

- Sketch the null clines for this system and determine the equilibrium solutions.

- For each equilibrium determine the type of the associated linearized solution and discuss whether or not the actual solution near the equilibrium behaves in the same way as does the linearized solution.
Question 5

Let a function \( f(t) \) have Laplace transform:

\[
\mathcal{L}(f)(s) = \frac{2}{s^3} + \frac{1}{s - 1} + \frac{3s + 4}{s^2 + 16}.
\]

- Determine the function \( f(t) \).
  Also give the appropriate range of the variable \( s \) for which \( f \) has the given Laplace transform.

- Determine the Laplace transform of \( f''(t) \).

- Determine the Laplace transform of \( e^{-2t} f(t) \).
Question 6

Using the method of Laplace transforms, solve the following differential equation:

\[ y' - 3y = 7e^{-4t}, \quad y(0) = 1. \]

Also discuss the behavior of the system as a function of time.