Question 1

Solve the differential equation:

\[ t \frac{dy}{dt} + 3y = \frac{4}{t^3}, \quad y(1) = -4. \]

Discuss the behavior of the solution forward and backward in time.
Question 2

Show that the following differential equation is exact and hence solve it:

$$(\sin(x) - 2y^2x^3)\,dx + (2y - yx^4)\,dy = 0.$$ 

Given the initial condition $y = 1$, when $x = 0$, discuss the behavior of the solution.
**Question 3**

Solve the following differential equation and discuss the behavior of the solution forward and backward in time:

\[
\frac{d^2y}{dt^2} - 12 \frac{dy}{dt} + 8y = 0, \quad \text{with initial conditions } y = -1 \text{ and } \frac{dy}{dt} = 2, \quad \text{when } t = 0.
\]
Question 4

A rocket shot vertically upwards on a certain planet experiences the deceleration of gravity of size 80 feet per second per second and a deceleration due to air resistance of size $\frac{v^2}{2000}$, where $v$ feet per second is the velocity of the rocket. If the rocket has initial velocity 3000 feet per second, find how high the rocket goes.
Question 5

Plot the slope field for the equation:

\[ \frac{dx}{dt} = (x - 1)(5 - x). \]

Discuss the behavior of the solutions and classify the equilibria.
Question 6

Consider the logistic equation with a constant harvesting rate $h$:

$$\frac{dx}{dt} = x(12 - x) - h.$$ 

Find the equilibria (if any) in the cases: $h = 32$, $h = 36$, $h = 40$.

Also discuss the stability of the equilibria.

In each case, if the initial population is $x = 7$, discuss what happens to the population as time passes.
Question 7

By row reducing an appropriate matrix find the value of the number $k$, such that the following linear system should have a solution and for that value of $k$ determine all the solutions:

\[
\begin{align*}
2x - y + 4z &= 13 \\
x - 3y + 7z &= 4 \\
4x + 3y - 2z &= k
\end{align*}
\]