Calculus III Quiz 8 11/18/5

Name: 
Signature: 
Show your work.

For this quiz, you will need the following indefinite integrals:

- \[ \int \cos^4(\theta) d\theta = \frac{1}{8}(3\theta + 3\sin(\theta) \cos(\theta) + 2\sin(\theta) \cos^3(\theta)) + C, \]
- \[ \int \cos^2(\theta) d\theta = \frac{1}{2}(\theta + \sin(\theta) \cos(\theta)) + C, \]
- \[ \int \sin^2(\theta) d\theta = \frac{1}{2}(\theta - \sin(\theta) \cos(\theta)) + C. \]

**Question 1**

Sketch the region between the two circles with the polar equations \( r = 2\cos(\theta) \) and \( r = 4\cos(\theta) \) and determine its centroid.

**Question 2**

- Sketch the ellipse with equation \( \frac{x^2}{9} + \frac{y^2}{25} = 1. \)
- Parametrize the ellipse.
- Find the line integral \( \int_\Gamma x \, dx - y \, dy \), where \( \Gamma \) is the ellipse, traced once counter-clockwise.
- Find the line integral \( \int_\Gamma x \, dy - y \, dx \), where \( \Gamma \) is the ellipse, traced once counter-clockwise.

**Question 3**

Find the line integral of \( \alpha = x \, dy - y \, dx \) taken over the triangle \( ABC \) in the plane with vertices \( A = [0, 0], B = [2, 0] \) and \( C = [1, 1] \), traced once counter-clockwise.