Calculus III Quiz 4 9/23/5

Name:  
Signature:  
Show your work.

Question 1
Let \( f(x, y) = x^2 + xy - 2y^2 - 8x + 5y + 7 \).

- Find the equation of the tangent plane to the surface \( z = f(x, y) \) at the point \((2, 3, -2)\).
- Find all points where the tangent plane to the surface \( z = f(x, y) \) is horizontal.
- Show that the level curve \( f(x, y) = 0 \) is a pair of lines (factorize the function \( f(x, y) \)) and sketch these lines.
- Explain why the other level curves of \( f(x, y) \) are hyperbolas.

Question 2
Let \( A = [4, -3, -4], \ B = [1, 3, 8] \) and \( C = [-1, 1, 6] \) and \( D = [2, 0, 5] \).

- Find the equation of the plane \( ABC \).
- Find the parametric equations of the line through \( D \) perpendicular to the plane \( ABC \).
- Find the distance of the point \( D \) from the plane \( ABC \).
- Find the mirror image of the point \( D \) in the plane \( ABC \).
- Find the volume of the tetrahedron through the points \( A, B, C \) and \( D \).