Honors Calculus I Quiz 4 9/23/5

Name: Signature:

Show your work.

Question 1
Let \( f(x) = x^2 + 4x - 5 \).
Show using the definition of the limit that the following limit is true:
\[
\lim_{x \to 2} f(x) = 7.
\]
Also determine an open interval \( J \) containing the point \( x = 2 \), such that for any \( x \) in \( J \) we have \(|f(x) - 7| < 0.5\).

Question 2
Let a differentiable function \( f(t) \) satisfy the properties: \( f'(t) > 0 \), \( f(2) = 4 \) and \( f''(2) = 3 \).

- Estimate the quantity \( f(2.5) \).
- Explain why the inverse function \( f^{-1}(t) \) exists and find the equation of the tangent line to the graph of the function \( y = f^{-1}(t) \) at \( t = 4 \).

Question 3
Let \( f(x) = x^3 - 8x^2 + 5x + 14 \).

- Find the intervals on which \( f'(x) > 0 \) and the intervals on which \( f'(x) < 0 \).
- Sketch the graphs of the functions \( y = f(x) \), \( y = f'(x) \) and \( y = f''(x) \) on the same sketch.
- Which is the largest open interval \( J \), containing the point \( x = 2 \), on which \( f \) has an inverse \( f^{-1} \)?
Explain your answer and give the domain \( K \) of the function \( f^{-1} \).
Also sketch the graph of the function \( f^{-1} \) for the domain \( K \).
- Find the equation of the tangent line to the graph of the function \( y = f^{-1}(x) \) at the point with \( x = 0 \).