Question 1

Let $f(x) = \frac{1}{x^2}$ and $g(x) = x^3 + 1$.

- What are the domains and ranges of $f$ and $g$?

- One of these maps is invertible and one is not. Which is which and why?

- Give formulas for the compositions $f \circ f$, $f \circ g$, $g \circ g$ and $g \circ f$ and obtain the derivatives of each of those compositions.

Question 2

Let $f'(t) = g(t)$ and $g'(t) = -f(t)$.

Prove that $f'^2 + g'^2 = C$ is constant.

If $f(0) = 0$ and $f'(0) = 1$, evaluate the constant $C$.

Also compute the derivative of the function $h(t) = \frac{f(t)}{g(t)}$.

Question 3

Find the linear approximation to the function $f(x) = (1 + 2x)^{-\frac{2}{3}}$ based at the origin and use it estimate the value of $f(0.1) = (1.2)^{-\frac{2}{3}}$.

By sketching the graphs of the function $f(x)$ and its linear approximation on the same graph, determine if your estimate is an under-estimate or an over-estimate.