Complex variables: Quiz 5 7/20/5

Name: Signature:
Show your work

Question 1

For each of the following series, find, with proof, the complex numbers $z$ for which the series converges and give a simple formula for the sum of the series:

$$f(z) = \sum_{n=0}^{\infty} \frac{z^n}{(2 - i)^n}$$

$$g(z) = \sum_{n=0}^{\infty} (n - 2)z^n$$

Hence find the sum of each of the following series, or explain why the sum in question does not exist:

$$A = \sum_{n=0}^{\infty} \frac{(1 + i)^n}{(2 - i)^n}$$

$$B = \sum_{n=0}^{\infty} (n - 2)\frac{(1 - i)^n}{(1 + i)^n}$$

$$C = \sum_{n=0}^{\infty} (n - 2)\frac{(1 - 2i)^n}{(3 + 4i)^n}$$

Question 2

Let $h(z) = \sum_{n=2}^{\infty} \frac{z^n}{n(n-1)}$.

- Find the radius of convergence of $h(z)$.
  Also show that the series for $h(z)$ converges for all $z$ on the boundary of its disc of convergence.

- Find the series for the first and second derivatives of $h(z)$.

- Hence express $h(z)$ in terms of standard functions.