Complex variables: Quiz 3 7/5/5

Name: Signature:
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Question 1
Consider the transformation \( z \to (1 + i)z + 2 + i \), defined for any \( z \in \mathbb{C} \).

- Describe the transformation geometrically.
- Find the fixed points of the transformation, if any.
- Find a formula for the inverse transformation.
- Find the images under the transformation of the lines \( \mathcal{L} \) and \( \mathcal{M} \) with the following parametric equations, where \( s \) and \( t \) are real parameters:

  \[ \mathcal{L} : z = 4 + (1 + i)t \quad \text{and} \quad \mathcal{M} : z = is. \]

Also sketch the lines \( \mathcal{L} \) and \( \mathcal{M} \) and their images on the same complex plane.
Verify that the angle between the lines \( \mathcal{L} \) and \( \mathcal{M} \) is the same as the angle between their respective image lines.

- Find the image under the transformation of the circle \( |z - 1 + i| = 3 \) and sketch the circle and its image on the complex plane.

Question 2
Let functions \( u(x, y) \) and \( v(x, y) \) be given by the formulas, for \( x \) and \( y \) real:

\[ u = x^2 - y^2 - 2x + 4y, \quad v = 2xy - 4x - 2y. \]

- Show that the pair \( (u, v) \) obeys the Cauchy-Riemann equations.
- Verify that \( u \) and \( v \) are harmonic functions.
- Show that the function \( u + iv \) may be expressed as a polynomial in the variable \( z = x + iy \).
- Hence factor the polynomial \( u^2 + v^2 \) as a product of linear factors in the variables \( x \) and \( y \).