Question 1

Sketch the following sets in the complex plane and for each identify whether the set is open, closed or neither and whether or not the set is connected. For each of these sets also give a parametrization or parametrizations of its boundary, as appropriate.

- \( A = \{ z : \Re(z) \geq \Im(z) \} \).
- \( B = \{ z : |z| < 1 \text{ and } |z - i| < 1 \} \).
- \( C = \{ z = re^{i\theta} : 1 < r < 2 \text{ and } -\frac{\pi}{3} \leq \theta \leq \frac{2\pi}{3} \} \).

Question 2

Find all complex numbers \( x \) such that \( x^6 = -64 \) and plot the solutions on the complex plane.

Question 3

Suppose that a real polynomial has a factor of \( x - \alpha \), where \( \alpha \) is a complex number that is not real. Prove that the polynomial has a real factor of the form \( x^2 - 2\Re(\alpha)x + |\alpha|^2 \). Verify that \( x = i \) is a root of the polynomial \( x^4 - 4x^3 + 6x^2 - 4x + 5 = 0 \) and hence find all its roots.