Integrated Calculus II Examination 2 Practice 2/25/5

Question 1

Calculate each of the following integrals.

- \[ \int_1^4 \left( \frac{3t^2 - 2t}{t^3 - t^2 + 1} \right) dt. \]

- \[ \int_1^9 x^2 \ln(x) \, dx. \]

- \[ \int_e^{e^2} \frac{(\ln(x))^2}{x} \, dx. \]

- \[ \int_0^1 2x^3 e^{-x^2} \, dx. \]

- \[ \int_0^{\pi/2} (\cos^5(x) - \cos^3(x)) \, dx. \]

- \[ \int_0^1 \frac{t - 10}{t^2 + 4t - 12} \, dt. \]

Question 2

Solve the following differential equations and discuss the behavior of each solution as a function of the variable \( t \).

Include a plot of each solution.

- \[ \frac{dy}{dt} = \frac{e^{3t}}{\sin(y)}, \quad y(0) = \frac{\pi}{2}. \]

- \[ \frac{dy}{dt} = 3(1 - y^2), \quad y(0) = \frac{1}{2}. \]
Question 3
Consider the following parametrized curve, describing the position of a wire in the plane:

\[ x = 12t^3 + 20t^2, \quad y = 15(t + 1)^2. \]

- The wire begins at the point \( A = (0, 15) \) and ends at the point \( B = (32, 60) \). Which \( t \)-values give these points?
- Sketch the wire \( AB \).
- Find the length of the wire \( AB \).
- Write integrals for the centroid of the wire.

Question 4
A region \( \mathcal{R} \) is bounded by the curves \( y = x^2 - 10x + 25 \) and \( y = -x^2 + 8x + 9 \).

- Sketch the region \( \mathcal{R} \).
- Find the area of the region \( \mathcal{R} \).
- If the region \( \mathcal{R} \) is rotated about the line \( x = 8 \), what is the volume of the solid of revolution that it generates?
- If the region \( \mathcal{R} \) is rotated about the \( x \)-axis, what is the volume of the solid of revolution that it generates?

Question 5
A lamina consists of two semi-circular flat metal discs \( A \) and \( B \), joined at one common point.

- Disc \( A \) is the upper half of the circular disc of radius 6 cm centered at \( (0, 6) \) (units in centimeters).
- Disc \( B \) is the right hand half of the circular disc of radius 6 cm centered at \( (6, 0) \).
- If the discs have equal density where is their center of mass?
- If disc \( A \) has density 20 grams per square centimeter and disc \( B \) has density 40 grams per square centimeter, where now is their center of mass?
Question 6
A water tank 10 meters long has a cross-section that is an isosceles triangle $ABC$, with $AB = BC$.
The edge $AB$ is level with water surface and is four meters long.
The vertex $C$ is three meters below the surface.
What is the volume of the tank?
How much work is done in pumping all the water out of the tank through a pipe that exits the tank level with the edge $AB$?
How much work is done in pumping all the water out of the tank through a pipe that exits five meters above the edge $AB$?

Question 7
A region $S$ is bounded by the curves $x = y^2 + 1$ and the line $x = 1 + 2y$.

- Sketch the region $S$.
- Find the area of the region $S$.
- Find the centroid of the region $S$.
- If the region $S$ is rotated about the $y$-axis, what is the volume of the solid of revolution that it generates?
- If the region $S$ is rotated about the $x$-axis, what is the volume of the solid of revolution that it generates?

Question 8
Find a potential function $P$ for each of the following exact differentials:

- $12x^3y\,dx + 3(x^4 + y^2)\,dy$.
- $8 \sin(x) \sin(2y)\,dx - 16 \cos(x) \cos(2y)\,dy$. 
**Question 9**

A hemispherical bowl of radius 10 meters is full of water (density 1000 kilograms per cubic meter).

How much work is done in removing the water through a pipe level with the surface of the bowl?

If instead the water is removed through pipe 5 meters above the level of the bowl, how much extra work is done?

**Question 10**

A bowl of soup is prepared at a temperature of 70 degrees Celsius in a room kept at 22 degrees Celsius.

After one minute the bowl has temperature 64 degrees Celsius.

The soup will be at a nice temperature to eat when its temperature is 48 degrees Celsius.

When will this be?

The soup will need to be reheated if the temperature drops below 37 degrees Celsius.

When will this be?

**Question 11**

The force \( F \) Newtons in a non-linear spring is given by \( F = kx^4 \) where \( k \) is a constant and \( x \) is the extension of the spring in meters from its rest length.

To produce an extension of 0.5 meters requires a force of 100 Newtons.

How much work is done in extending the spring from \( x = 0.5 \) to \( x = 1 \) meter.