Integrated Calculus II Examination 2 3/2/5

Each problem is worth 25 points.
The best four problems count.
Show your work

Question 1
Calculate each of the following integrals.

- \[ \int_{-1}^{3} \frac{t - 1}{t^2 - 2t - 8} \, dt. \]

- \[ \int_{0}^{\pi/3} (\sin^5(x) - \sin^3(x)) \, dx. \]

- \[ \int_{0}^{1} 12x^3 e^{2x^2} \, dx. \]
**Question 2**

Solve the following differential equation and discuss the behavior of the solution as a function of the variable $t$.

Include a plot of the solution.

\[
\frac{dy}{dt} = \frac{2\cos(2t)(1 + y^2)}{y}, \quad y(0) = 4.
\]
Question 3

Daniel and Giorgiana buy caffe lattes at the Eurostar Cafe, whose room temperature is kept at 25 degrees Celsius,

- The caffe lattes are prepared and delivered to Daniel and Giorgiana at a temperature of 80 degrees Celsius.

- After one minute the lattes cool to 70 degrees Celsius.

- Customer Giorgiana likes to drink her latte when its temperature is about 55 degrees. How soon after the caffe is served will she be able to drink it?

- Customer Daniel likes to drink his latte, when its temperature is a cool 40 degrees. How much extra time must he wait after Giorgiana has started drinking her latte, before he starts drinking his own latte?
Question 4

A lamina consists of a semi-circular flat metal disc $A$ and a right-angled triangle, $B$ joined along a side of the triangle and the straight (diameter) edge of the disc.

- Disc $A$ is the upper half of the circular disc of diameter 10 cm centered at the point $(5, 0)$ (units in centimeters).
- Triangle $B$ has one vertex at the origin, another at the point $(10, 0)$ and the third at $(0, -20)$.
- Sketch the lamina.
- If the disc and the triangle have equal density where is the center of mass of the lamina?
- If disc $A$ has density 15 grams per square centimeter and triangle $B$ has density 30 grams per square centimeter, where now is the center of mass of the lamina?
Question 5

A region $S$ is bounded by the parabola $y = x^2 - 6x + 9$ and the line $y = 5 - x$.

- Sketch the region $S$.
- Find the area of the region $S$.
- If the region $S$ is rotated about the $y$-axis, what is the volume of the solid of revolution that it generates?
- If the region $S$ is rotated about the $x$-axis, what is the volume of the solid of revolution that it generates?
Question 6

A hemispherical bowl of radius 12 meters is full of water (density 1000 kilograms per cubic meter).

- How much work is done in removing the water through a pipe level with the surface of the bowl?

- If instead the water is removed through a pipe 6 meters below the level of the bowl, how much work is done?