

**SCHEDULE:** Homework from Week 2 is due **Friday, September 12.**

- Wednesday, September 3rd: Section 1.3
- Friday, September 5th: Section 1.4
- Monday, September 8th: start Section 1.5 - eigenvalues/vectors, diagonalization, implications for symmetric matrices
- Wednesday, September 10th: LAB in GSCC 126

**TOPICS:**

- *Section 1.3: Positive Definite Matrices and  $LDL^T$*  - Positive definite matrices are a certain type of symmetric matrices that have some special properties and show up in a variety of applications. In particular, in this section we explore the connection between Gaussian elimination and completing the square to determine whether a quadratic function has a minimum. It turns out that a minimum exists exactly when a matrix related to the function is positive definite.

**homework:** pg. 28-30, 1.3.1, 1.3.5, 1.3.9, 1.3.11.

- *Section 1.4: Minimum Principles* - It turns out that for  $A$  positive definite, the quadratic function  $P(x) = \frac{1}{2}x^T Ax - x^T b$  is minimized at  $x$  such that  $Ax = b$ . This fact is used to arrive at something called the *least squares solution* of an equation  $Ax = b$  that does not have an exact solution.

**homework:** pg. 45-46, 1.4.1, 1.4.2, 1.4.4.

- *Section 1.5: Eigenvalues and Dynamical Systems* - Multiplication of a vector by a matrix in general will change the direction of the vector, unless the vector is an *eigenvector* of the matrix. In that case, matrix multiplication preserves direction (up to possible reversal by  $180^\circ$ ) and simply stretches the vector by a factor, called the *eigenvalue* associated with the eigenvector. Eigenvalues  $\lambda$  of  $A$  can be found by solving the characteristic polynomial  $\det(A - \lambda I) = 0$ , with eigenvectors  $x$  satisfying  $Ax = \lambda x$ . If  $A \in \mathbb{M}^{n \times n}$  has  $n$  linearly independent eigenvectors, then they can be used to *diagonalize*  $A$  - that is, to factor  $A = S\Lambda S^{-1}$  for  $\Lambda$  diagonal. If  $A$  is symmetric, then its eigenvalues are real and its eigenvectors are orthogonal to each other. Eigenvalues and eigenvectors are useful in a variety of contexts, including solution of linear ordinary differential equations, which we will come to next week.

**homework:** pg. 64-67, 1.5.1, 1.5.2, 1.5.9, 1.5.11, 1.5.23.

**GRADED ASSIGNMENT: Due Friday, September 12th.** Some of these exercises are best done using Matlab. Be sure to get as far as you can on these, and then bring your work to Lab in GSCC on Wednesday, September 10th. You will also be given additional exercises on that day, also due September 12.

1. The following graph shows the flow of resources that a company forecasts. The lettered nodes represent industries that the company runs, while the numbers and variables along the edges of the graph represent monetary value of inputs and outputs.

- a) Write down a linear system of equations that describes the flow of resources through each node and give the corresponding augmented matrix.
  - b) Show that there are 2 degrees of freedom in the system and write down equations expressing  $x_1, x_2, x_3, x_4$  in terms of  $x_5, x_6$ .
  - c) Assuming of course that  $x_j \geq 0$  for all  $j = 1 \dots 6$ , use your equations to find upper bounds on  $x_5, x_6$ .
  - d) Industries A and B operates less cleanly than D, such that the company is penalized according to the size of  $x_1, x_2, x_3$  and  $x_4$ . Using parts b) and c), write down a solution to this system that minimizes  $C = x_1 + x_2 + x_3 + x_4$ , subject to  $x_j \geq 0$  for all  $j = 1 \dots 6$ .
2. pg. 28, 1.3.7. Also, write out  $A'$  as a sum of matrices, as in the example on pg. 21-2, find the corresponding quadratic function  $f(x_1, x_2, x_3)$ , and use  $A'$  to write  $f$  as a sum of squares.
  3. pg. 45, 1.4.7.
  4. pg. 45, 1.4.8.
  5. pg. 66, 1.5.22.
  6. Suppose that in one county, the registered voters are distributed as 150,000 independents, 90,000 Democrats, and 60,000 Republicans. Suppose that 70% of independents remain independents each year, 20% become Democrats, and 10% become Republicans; 20% of Democrats become independents, 70% remain Democrats, and 10% become Republicans; while 10% of Republicans become independents, 10% become Democrats, and 80% become Republicans. Determine the numbers of voters in each group in the long run.