Weeks 7-8 **Math 1530** *Advanced Calculus 1* - RUBIN - Fall '03

SCHEDULE: Homework from Weeks 7-8 is due in class on **Wednesday, October 22**.

- Wednesday, October 8: finish Section 3.2, Section 3.3
- Friday, October 10: review
- Monday, October 13: Midterm Exam on all material up to and including Section 3.2.
- Wednesday, October 15: Section 3.4
- Friday, October 17: Section 3.5
- Monday, October 20: Section 4.1

TOPICS:

Section 3.3: Nested Set Property - The nested set property is a particular application of the idea of compactness. In various settings, sets are defined, via an iterative process, such that each is compact (e.g. closed and bounded in $\mathbb{R}^n$) and nonempty and each subsequent set lies inside the previous one. The nested set property then implies that the "limit" of such a process will consist of a nonempty set.

Reading Objectives: After reading Section 3.3, students should be able to:

- State the nested set property, and give an idea of how it follows from the compactness of the nested sets, as well as why compactness is necessary.

homework: NOTE: The first two do not require the nested set property.

1. Consider the following diagram. Each arrow represents an implication which may or may not hold. To each, there corresponds a number. List whether each implication is true or false, and give a reason for each. If the implication is covered by a result in the book, then just give the reference to the result in the book. If not, then give a proof or counterexample, as appropriate. For example, for numbers 1 and 2, you can state: 1. True - Theorem 3.1.3. 2. True - Theorem 3.1.3.

2. pg. 176, # 39a-d (see # 14 on pg. 174 for the definition of isolated).

Section 3.4: Path-Connected Sets - Loosely speaking, a set is path connected if for any two points in the set, there exists a continuous curve between the two that does not leave the set.

This is a strong version of connectedness; we will encounter a weaker version in the next section.

Reading Objectives: After reading Section 3.4, students should be able to:

- Understand the concepts of a *continuous path* and of a *path-connected set.*
- Determine whether a set, such as $\{(x, y) \in \mathbb{R}^2 : 1 \le x^2 + y^2 \le 3\}$ or $\mathbf{Q} \cap [0, 1]$, is path-connected.

Section 3.5: Connected Sets - Loosely speaking, a connected set is one that cannot be separated into disjoint components contained in distinct open neighborhoods. One reason that path-connectedness is useful is that it implies connectedness (Theorem 3.5.2); often the easiest way to prove that a set is connected is to show that it is path-connected. Not every connected set is path-connected, however.

Reading Objectives: After reading Section 3.5, students should be able to:

- Understand the rigorous definitions of *separation* of a subset of a metric space by open sets and of a *connected set.*
- Give an example of a connected set that is not path-connected.
- Determine whether a set, such as $\{(x, y) \in \mathbb{R}^2 : 1 \le x^2 + y^2 \le 3\}$ or $\mathbf{Q} \cap [0, 1]$, is connected.

homework for 3.4 and 3.5:

1. Determine (by proof or counterexample) the truth or falsity of the following statements:

   a) (A is compact in $\mathbb{R}^n$) $\Rightarrow$ (A is connected in $\mathbb{R}^n$).
   b) ($\mathbb{R}^n \setminus A$ is connected) $\Rightarrow$ (A is connected in $\mathbb{R}^n$).
   c) ($A \subset \mathbb{R}^3$ is connected) $\Rightarrow$ (A is connected as a subset of $\mathbb{R}^4$)
   d) ($A = \{x \in \mathbb{R}^n : ||x|| \le 1\}$) $\Rightarrow$ ($\mathbb{R}^n \setminus A$ is connected) NOTE: Consider $n = 1$ and $n \ge 2$ separately.

2. pg. 175, #24.
3. pg. 175, #28.

Section 4.1: Continuous Mappings - It is finally time to focus on functions (on general metric spaces). In this section we define the limit of a function at a point and what it means for a function to be continuous (at a point or on a set). The main result in this section is that there are four equivalent ways to define continuity of a function, one in terms of epsilons and deltas, one in terms of converging sequences, and two that are topological.

Reading Objectives: After reading Section 4.1, students should be able to:

- Determine at what points the limit of a function does not exist.
- Test whether a function such as $f : \mathbb{R} \to \mathbb{R}, x \mapsto x^2$ is continuous at a fixed point x_0.
- List four equivalent ways to define continuity.

homework: To come...