SCHEDULE: Homework from Week 6 is due in class on **Wednesday, October 8**.

- Wednesday, October 1: finish Section 2.9
- Friday, October 3: start Section 3.1
- Monday, October 6: finish Section 3.1, Section 3.2

TOPICS:

Section 2.9: Series of Real Numbers and Vectors - A series is a sum of a countably infinite number of terms. We define convergence of a series in terms of a convergence of the sequence of partial sums of the series; the nth term in this sequence consists of the sum of the first n terms of the series. This section presents a variety of tests that can be used to check whether a series converges or diverges (fails to converge); see Theorem 2.9.4. Theorem 2.9.2 states an important result relating convergence of a series to bounding of sums of the terms far enough out in the series.

Reading Objectives: After reading Section 2,9, students should be able to:

- Given a series, define the sequence of partial sums corresponding to that series.
- State two (equivalent) criteria for convergence of a series (see Theorem 2.9.2).
- Define absolute convergence and explain how absolute convergence relates to convergence.
- Understand the general ideas used to prove Theorem 2.9.4 on convergence tests, especially the first 6 parts.
- Use these convergence tests to check the convergence of given series, such as $\sum\limits_{n=1}^{\infty} \frac{(\sin n)^n}{n^2}$, $\sum\limits_{n=1}^{\infty} \frac{n^2}{4^n}$, or $\sum\limits_{n=1}^{\infty} \frac{2^n+n}{3^n-n}$.

homework:

1. pg. 148, # 47.
2. pg. 148, # 48a.
3. pg. 149, # 52a,c.
4. Test the series $\sum_{n=1}^{\infty}(n^{1/n}-1)^n$ for convergence. Use the result to test the series $\sum_{n=1}^{\infty} n^{-1-(1/n)}$ for convergence.
5. If $\sum_{n=1}^{\infty} a_n$ converges absolutely, prove that $\sum_{n=1}^{\infty} a_n^2$ converges (Hint: use Theorem 2.9.2). Is the converse true?

Section 3.1: Compactness - A set is compact, roughly speaking, if it can be contained in a finite number of small open sets. This is a very useful concept, in part because compactness is equivalent to sequential compactness: in a compact set, every sequence has a convergent subsequence. There are many different concepts relating to compactness, including total boundedness and the finite intersection property. It is important to keep straight how all of these relate, as well as how these things relate to other properties of sets, such as being closed or bounded.

Reading Objectives: After reading Section 3.1, students should be able to:

- Define what it means for a set to be compact, sequentially compact, totally bounded, or to have the finite intersection property and state the relationships among these properties.
- Use these concepts, and their interrelations, to check whether specific sets, such as $\{(x, y) \in \mathbb{R}^2 : 0 \leq x < 1, 0 \leq y \leq 1\}$, are compact.

Section 3.2: Heine-Borel Theorem - The Heine-Borel Theorem states that in $\mathbb{R}^n$, compactness is equivalent to being closed and bounded. We already know from Section 3.1 that a compact set is always closed and bounded, in any metric space. The Heine-Borel Theorem can be proved using at least two approaches, one involving sequential compactness and one involving direct consideration of an arbitrary open cover of a closed, bounded set in $\mathbb{R}^n$.

Reading Objectives: After reading Section 3.2, students should be able to:

- Use the Heine-Borel Theorem to check for compactness of subsets of $\mathbb{R}^n$, such as $\{(x, y) \in \mathbb{R}^2 : 0 \leq x < 1, 0 \leq y \leq 1\}$.
- Understand how the Bolzano-Weierstrass Theorem and sequential compactness can be used to prove the Heine-Borel Theorem.
- Outline the inductive steps used to prove the Heine-Borel Theorem using direct consideration of an arbitrary open cover of a closed, bounded set in $\mathbb{R}^n$.

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homework for 3.1 and 3.2:

1. pg. 172, # 1a,c,d,e,g,h,i (don't worry about connectedness, and just give a brief justification of your answer for each)
2. pg. 174, # 17.
3. pg. 174, # 20.