SCHEDULE: Homework (and bonus) from Week 1 is due **Wednesday, September 5**.

- Monday, August 25th: Introduction
- Wednesday, August 27th: Section 1.2
- Friday, August 29th: finish Section 1.2, Section 1.3
- Monday, September 1st: NO CLASS - LABOR DAY
- Office hours are canceled on Monday, September 1st. Feel free to make an appointment if you'd like to see me between 8/29 and 9/3.

TOPICS:

Section 1.2: Completeness and the Real Number System - In this section, the rational numbers are extended to the larger set of real numbers by collecting all limits of convergent sequences of rationals. We will not emphasize this but rather will use this opportunity to remind ourselves about convergence of sequences. Useful properties in this context include boundedness and monotonicity.

Reading objectives: After reading Section 1.2, students should be able to:

- Define a sequence and give an example of a sequence that has any subset of the following properties: *convergence*, *boundedness*, and *monotonicity*.
- Explain which of the above properties, or combinations of properties, imply which of the other properties, or combinations of properties (e.g. a convergent sequence is always bounded but not vice versa - why, in heuristic terms?).
- Use the above properties to prove the convergence of a sequence such as $x_n = 1/3^n$.
- Define what it means for a set to be *dense* in another set.

homework: Define a sequence recursively by $x_0 = 1/2$, $x_n = 1 - \sqrt{1 - x_{n-1}}$ for $n \geq 1$.

a) Does this sequence converge? Justify your answer fully.

b) Suppose $x_0 \in [0, 1]$. Does this sequence converge? If so, find the limit of the sequence. (HINT: It may be useful to keep in mind that for a sequence x_n of nonnegative real numbers with nonnegative limit x, we have $\sqrt{x_n} \to \sqrt{x}$.)

Section 1.3: Least Upper Bounds - In this section, the main result is that every nonempty subset of $\mathbb{R}$ with an upper bound has a least upper bound (*supremum*). Similarly, every nonempty subset of $\mathbb{R}$ with a lower bound has a greatest lower bound (*infimum*).

Reading Objectives: After reading Section 1.3, students should be able to:

- Define upper and lower bounds of a subset of $\mathbb{R}$ (nonempty or empty).
- Distinguish between an upper bound and a least upper bound.
- Find the supremum and infimum of a set such as $\{x \in \mathbb{R} : x^2 < 5\}$ or $\{x \in \mathbb{R} : x^2 > 7\}$.

homework: Pg. 97-8, # 4,5,7,8. At first, be sure to try #4 without referring to your class notes or book.

BONUS: Let $x_0 = 0$ and $x_1 = 1$. Define a sequence in $\mathbb{R}$ by setting

$$x_{n+1} = \frac{1}{n+1} x_{n-1} + \frac{n}{n+1} x_n, \; n \geq 1$$

Does this sequence converge? If so, what is the limit?