Section 6.1

3. We have $t_0 = 0$, $y_0 = 1$ and $f(t, y) = ty$. Thus, the first step of Euler's method is completed as follows.

$$y_1 = y_0 + t_0 y_0 h = 1 + 0 \times 1 \times 0.1 = 1.0$$
$$t_1 = t_0 + h = 0 + 0.1 = 0.1.$$

The second step follows.

$$y_2 = y_1 + t_1 y_1 h = 1.0 + 0.1 \times 1 \times 0.1 = 1.01$$
$$t_2 = t_1 + h = 0.1 + 0.1 = 0.2.$$

Continuing in this manner produces the results in the following table.

k	t_k	y_k	$f(t_k, y_k) = t_k y_k$	h	$f(t_k, y_k)h$
0	0.0	1.0000	0.0000	0.1	0.0000
1	0.1	1.0000	0.1000	0.1	0.0100
2	0.2	1.0100	0.2020	0.1	0.0202
3	0.3	1.0302	0.3091	0.1	0.0309
4	0.4	1.0611	0.4244	0.1	0.0424
5	0.5	1.1036	0.5518	0.1	0.0552

9. The equation $z' = (1 + t)z$ is separable.

$$\frac{dz}{z} = (1 + t)\, dt$$
$$\ln|z| = t + \frac{1}{2}t^2 + C$$

The initial condition $z(0) = -1$ produces $C = 0$ and $\ln|z| = t + (1/2)t^2$. Of course, we choose the negative branch.

$$|z| = e^{t+(1/2)t^2}$$
$$z = -e^{t+(1/2)t^2}$$

In the figure, three numerical solutions and the exact solution are pictured. The numerical solutions were calculated using Euler's method and step sizes $h = 0.2$, $h = 0.1$, and $h = 0.05$.