**Section 3.1**

3. Recall the equation of the Malthusian model $P(t) = Ce^{rt}$. The constant $C$ is the initial population $P(0)$, so the tripling condition is stated as $P(10) = P(0)e^{10r} = 3P(0)$. Thus, $r = (1/10)\ln 3 \approx 0.1099$. The population doubles precisely when $P(t) = 2P(0) = P(0)e^{(t\ln 3)/10}$, that is, when $(t\ln 3)/10 = \ln 2$. Solving, one obtains $t = (10\ln 2)/\ln 3 \approx 6.3093$.

10. To find the minimum and maximum growth rate, use the first derivative test on the growth rate $dP/dt$. That is, set $d^2P/dt^2 = 0$. Obtain

$$\frac{d^2P}{dt^2} = rP'\left(1 - \frac{2P}{k}\right).$$

Thus, $dP/dt$ reaches an extremum when $P' = 0$ and when $P = k/2$. One wants to show that $dP/dt$ is a maximum when $P = k/2$. So, one checks that $d^2P/dt^2$ changes sign from positive to negative at $P = k/2$. When $P = k/2$, $P' = r(k/2)(1 - (k/2)/k) = rk/4$, which is positive. So any possible change in sign of $d^2P/dt^2$ is in the third factor $(1 - 2P/k)$. This factor does change sign from positive to negative at $P = k/2$, so $d^2P/dt^2$ changes sign from positive to negative at $P = k/2$. Therefore $dP/dt$ reaches a maximum at $P = k/2$.

12. The carrying capacity $K = 20,000$, and the initial condition $P_0 = 1000$, and it is given that $P(10) = 2000$. Using equation (1.13), one obtains

$$2000 = \frac{(20000)(1000)}{1000 + (19000)e^{-10r}}.$$

Solving gives $r = -(\ln(9/19))/10 \approx 0.0747$. After 25 hours, the population is

$$P(25) = \frac{(20000)(1000)}{1000 + (19000)e^{2.5\ln(9/19)}} \approx 5084.$$

Now, find $t$ so that $P(t) = K/2 = 10000$. This gives

$$e^{\frac{1}{10}\ln(\frac{9}{19})t} = \frac{2000 - 1000}{19000}.$$

That is, $t = 10(-\ln(19))/(\ln(9/19)) \approx 39.4055$.

16. (a) Since the population is measured in thousands, the fishing rate is $h = 0.1$ thousand fish per day. The modified model is $P' = 0.1P(1 - P/10) - 0.1$.

(b) We look for roots of $0.1P(1-P/10) - 0.1 = 0$. This leads to the quadratic equation $P^2 - 10P + 10 = 0$, which has roots $P = 5 \pm \sqrt{15}$. Using the graph of $0.1P(1 - P/10) - 0.1$ we see that $P_1 = 5 - \sqrt{15} \approx 1.127$ is an unstable equilibrium point, and $P_2 = 5 + \sqrt{15} \approx 8.873$ is an asymptotically stable equilibrium point.

(c) We have $P' = 0.1P(1 - P/10) - 0.1 > 0$ for $P_1 < P < P_2$ and negative elsewhere. It follows using qualitative analysis that for any starting population greater than $P_1$, the population tends to the stable equilibrium at $P_2$. For any starting population smaller than $P_1$, the population decreases until it dies out. In particular a population of 1000 is doomed while a population of 2000 tends to $P_2$.

**Section 3.3**

3. Let $P(t)$ represent the balance in the account $t$ years after the initial investment. Let $r$ represent the annual rate, $d$ the yearly deposit, and $P_0$ the initial investment. Then,

$$P' = rP + d, \quad P(0) = P_0.$$

This equation is linear, with integrating factor $e^{-rt}$. Consequently,

$$\left(e^{-rt}P\right)' = de^{-rt},$$
$$e^{-rt}P = -\frac{d}{r}e^{-rt} + C,$$
$$P = -\frac{d}{r} + Ce^{rt}.$$

Use $P(0) = P_0$ to produce $C = P_0 + d/r$ and

4. Let $P(t)$ represent the balance in the account $t$ years after Jason's day of birth. Let $r$ represent the annual rate, $d$ the annual deposit. Since no initial investment is required,

$$P' = rP + d, \quad P(0) = 0.$$

This equation is linear, with integrating factor $e^{-rt}$. Consequently,

$$\left(e^{-rt}P\right)' = de^{-rt},$$
$$e^{-rt}P = -\frac{d}{r}e^{-rt} + C,$$
$$P = -\frac{d}{r} + Ce^{rt}.$$

Use $P(0) = 0$ to produce $C = d/r$ and

$$P(t) = \frac{d}{r}\left(e^{rt} - 1\right).$$

Because $P(18) = 50000$,

$$50000 = \frac{d}{r}\left(e^{r(18)} - 1\right),$$
$$d = \frac{50000r}{e^{18r} - 1},$$
$$d = \frac{50000(0.0625)}{e^{18(0.0625)} - 1},$$
$$d \approx \$1,502.25$$

5. Let $P(t)$ represent the balance in the account after $t$ years. Let $r$ represent the annual rate, $w$ the yearly withdrawal, and $P_0$ the amount of the inheritance. Then

$$P' = rP - w \quad P(0) = P_0.$$

The equation is linear with integrating factor $e^{-rt}$. Consequently,

$$\left(e^{-rt}P\right)' = -we^{-rt},$$
$$e^{-rt}P = \frac{w}{r}e^{-rt} + C,$$

Use $P(0) = P_0$ to produce $C = P_0 - w/r$ and

$$P(t) = \frac{w}{r} + \left(P_0 - \frac{w}{r}\right)e^{rt}.$$

Now, to find when the funds are depleted, set $P(t) = 0$.

$$0 = \frac{w}{r} + \left(P_0 - \frac{w}{r}\right)e^{rt},$$
$$e^{rt} = \frac{w/r}{w/r - P_0},$$
$$t = \frac{1}{r}\ln\frac{w/r}{w/r - P_0}.$$

Thus, the account will be depleted in

$$t = \frac{1}{0.05}\ln\frac{8000/0.05}{8000/0.05 - 50000} \approx 7.5 \text{ years}.$$

**Section 3.4**

4. The model equation is $Q' + Q/2 = 5\cos 3t$. The solution is

11. The model equation is $I' + I/10 = 10 - 2t$. The solution is

$$I(t) = 300 - 20t - 300e^{-t/10}.$$