1.1 Exercises

In Exercises 1–6, refer to the accompanying figure and determine the coordinates of the point and the quadrant in which it is located.

1. A
2. B
3. C
4. D
5. E
6. F

In Exercises 7–12, refer to the accompanying figure.

7. Which point is represented by the ordered pair (4, 2)?
8. What are the coordinates of point B?
9. Which points have negative y-coordinates?
10. Which point has a negative x-coordinate and a negative y-coordinate?
11. Which point has an x-coordinate that is equal to zero?
12. Which point has a y-coordinate that is equal to zero?

In Exercises 13–20, plot the point on a set of coordinate axes.

13. (−2, 5)
14. (1, 3)
15. (3, −1)
16. (3, −4)

17. \((8, −\frac{7}{2})\)
18. \((-\frac{3}{2}, \frac{3}{4})\)
19. \((4.5, −4.5)\)
20. \((1.2, −3.4)\)

In Exercises 21–24, find the distance between the points.
21. \((2, 5)\) and \((5, 9)\)
22. \((1, 0)\) and \((4, 4)\)
23. \((-1, 3)\) and \((4, 9)\)
24. \((-3, 2)\) and \((9, 7)\)

25. Find the coordinates of the points that are 10 units away from the origin and have a y-coordinate equal to −6.
26. Find the coordinates of the points that are 5 units away from the origin and have an x-coordinate equal to 3.
27. Show that the points \((3, 4), (−3, 7), (−6, 1),\) and \((0, −2)\) form the vertices of a square.
28. Show that the triangle with vertices \((-5, 2), (−2, 5),\) and \((5, −2)\) is a right triangle.

In Exercises 29–34, find an equation of the circle that satisfies the given conditions.
29. Radius 4 and center \((1, −2)\)
30. Radius 3 and center \((-2, −4)\)
31. Radius 5 and center at the origin
32. Center at the origin and passes through \((2, 3)\)
33. Center \((2, −3)\) and passes through \((5, 2)\)
34. Center \((-a, a)\) and radius \(2a\)

35. Tracking a Criminal with GPS After obtaining a warrant, the police attached a GPS tracking device to the car of a murder suspect. Suppose the car was located at the origin of a Cartesian coordinate system when the device was attached. Shortly afterwards, the suspect’s car was tracked going 5 mi due east, 4 mi due north, and 1 mi due west before coming to a permanent stop.

a. What are the coordinates of the suspect’s car at its final destination?
b. What was the distance traveled by the suspect?
c. What is the distance as the crow flies between the original position and the final position of the suspect’s car?
36. **Planning a Grand Tour** A grand tour of four cities begins at City A and makes successive stops at Cities B, C, and D before returning to City A. If the cities are located as shown in the accompanying figure, find the total distance covered on the tour.

![Diagram of cities A, B, C, and D with distances]

37. **Will You Incur a Delivery Charge?** A furniture store offers free setup and delivery services to all points within a 25-mi radius of its warehouse distribution center. If you live 20 mi east and 14 mi south of the warehouse, will you incur a delivery charge? Justify your answer.

38. **Optimizing Travel Time** Towns A, B, C, and D are located as shown in the accompanying figure. Two highways link Town A to Town D. Route 1 runs from Town A to Town D via Town B, and Route 2 runs from Town A to Town D via Town C. If a salesman wishes to drive from Town A to Town D and traffic conditions are such that he could expect to average the same speed on either route, which highway should he take to arrive in the shortest time?

![Diagram of routes A-B-D and A-C-D]

39. **Optimizing Travel Time** Refer to Exercise 38. Suppose that the salesman's average speeds from Town A to Town B and from Town B to Town D along Route 1 are 45 mph and 60 mph, respectively. Also, suppose that his average speeds from Town A to Town C and from Town C to Town D along Route 2 are 75 mph and 50 mph, respectively. Which highway should he take to arrive at his final destination in the shortest possible time? How much time does he save?

![Diagram of speeds and distances]

40. **Minimizing Shipping Costs for a Fleet of Autos** Refer to the figure for Exercise 38. Suppose a fleet of 100 automobiles are to be shipped from an assembly plant in Town A to Town D. They may be shipped either by freight train along Route 1 at a cost of $66e/mile/automobile or by truck along Route 2 at a cost of $62e/mile/automobile. Which means of transportation minimizes the shipping cost? What is the net savings?

41. **Cost of Laying Cable** In the accompanying diagram, S represents the position of a power relay station located on a straight coastal highway, and M shows the location of a marine biology experimental station on a nearby island. A cable is to be laid connecting the relay station at S with the experimental station at M via the point Q that lies on the x-axis between O and S. If the cost of running the cable on land is $300/running foot and the cost of running cable underwater is $500/running foot, find an expression in terms of x that gives the total cost of laying the cable. Use this expression to find the total cost when x = 1500 and when x = 2500.

![Diagram of cable laying]

42. **Purchasing an HDTV Antenna** Will Barclay wishes to determine which HDTV antenna he should purchase for his home. The TV store has supplied him with the following information:

<table>
<thead>
<tr>
<th>Range in Miles</th>
<th>VHF</th>
<th>UHF</th>
</tr>
</thead>
<tbody>
<tr>
<td>30</td>
<td>20</td>
<td>A</td>
</tr>
<tr>
<td>45</td>
<td>35</td>
<td>B</td>
</tr>
<tr>
<td>60</td>
<td>40</td>
<td>C</td>
</tr>
<tr>
<td>75</td>
<td>55</td>
<td>D</td>
</tr>
</tbody>
</table>

Will wishes to receive Channel 17 (VHF), which is located 25 mi east and 35 mi north of his home, and Channel 38 (UHF), which is located 20 mi south and 32 mi west of his home. Which model will allow him to receive both channels at the least cost? (Assume that the terrain between Will's home and both broadcasting stations is flat.)

43. **Distance Between Two Cruise Ships** Two cruise ships leave port at the same time. Ship A sails north at a speed of 20 mph while Ship B sails east at a speed of 30 mph. Find an expression in terms of the time t (in hours) giving the distance between the two cruise ships.
b. Using the expression obtained in part (a), find the distance between the two cruise ships 2 hr after leaving port.

44. **Distance Between Two Cargo Ships** Sailing north at a speed of 25 mph, Ship A leaves a port. A half hour later, Ship B leaves the same port, sailing east at a speed of 20 mph. Let $t$ (in hours) denote the time Ship B has been at sea.

a. Find an expression in terms of $t$ that gives the distance between the two cargo ships.

b. Use the expression obtained in part (a) to find the distance between the two cargo ships 2 hr after Ship A has left the port.

45. **Watching a Rocket Launch** At a distance of 4000 ft from the launch site, a spectator is observing a rocket being launched. Suppose the rocket lifts off vertically and reaches an altitude of $x$ feet, as shown below:

![Rocket Launch Diagram]

a. Find an expression giving the distance between the spectator and the rocket.

b. What is the distance between the spectator and the rocket when the rocket reaches an altitude of 20,000 ft?

46. a. Show that the midpoint of the line segment joining the points $P_1(x_1, y_1)$ and $P_2(x_2, y_2)$ is

$$
\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)
$$

b. Use the result of part (a) to find the midpoint of the line segment joining the points $(-3, 2)$ and $(4, -5)$.

47. **A Scavenger Hunt** A tree is located 20 yd to the east and 10 yd to the north of a house. A second tree is located 10 yd to the east and 40 yd to the north of the house. The prize in a scavenger hunt is placed exactly midway between the trees.

a. Place the house at the origin of a Cartesian coordinate system, and draw a diagram depicting the situation.

b. What are the coordinates of the position of the prize?

c. How far is the prize from the house?

48. Find an equation of a circle given that $(3, -5)$ and $(-1, -3)$ are the endpoints of its diameter.

In Exercises 49 and 50, determine whether the statement is true or false. If it is true, explain why it is true. If it is false, give an example to show why it is false.

49. If the distance between the points $P_1(a, b)$ and $P_2(c, d)$ is $D$, then the distance between the points $P_1(a, b)$ and $P_3(kc, kd)$ ($k \neq 0$) is given by $|k|D$.

50. The circle with equation $kx^2 + ky^2 = a^2$ lies inside the circle with equation $x^2 + y^2 = a^2$, provided that $k > 1$ and $a > 0$.

51. Let $(x_1, y_1)$ and $(x_2, y_2)$ be two points lying in the $xy$-plane. Show that the distance between the two points is given by

$$
d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}
$$

**Hint:** Refer to the accompanying figure, and use the Pythagorean Theorem.

52. In the Cartesian coordinate system, the two axes are perpendicular to each other. Consider a coordinate system in which the $x$-axis and $y$-axis are noncollinear (that is, the axes do not lie along a straight line) and are not perpendicular to each other (see the accompanying figure).

a. Describe how a point is represented in this coordinate system by an ordered pair $(x, y)$ of real numbers. Conversely, show how an ordered pair $(x, y)$ of real numbers uniquely determines a point in the plane.

b. Suppose you want to find a formula for the distance between two points, $P_1(x_1, y_1)$ and $P_2(x_2, y_2)$, in the plane. What advantage does the Cartesian coordinate system have over the coordinate system under consideration?
In Exercises 5–10, find the slope of the line that passes through the given pair of points.

5. (4, 3) and (7, 8)  
6. (4, 5) and (3, 8)  
7. (−2, 3) and (4, 8)  
8. (−2, −2) and (4, −4)  
9. (a, b) and (c, d)  
10. (−a + 1, b − 1) and (a + 1, −b)

11. Given the equation \( y = 2x - 3 \), answer the following questions.
   a. If \( x \) increases by 1 unit, what is the corresponding change in \( y \)?
   b. If \( x \) decreases by 2 units, what is the corresponding change in \( y \)?

12. Given the equation \( 2x + 3y = 4 \), answer the following questions.
   a. Is the slope of the line described by this equation positive or negative?
   b. As \( x \) increases in value, does \( y \) increase or decrease?
   c. If \( x \) decreases by 2 units, what is the corresponding change in \( y \)?

In Exercises 13–18, match the statement with one of the graphs (a)–(f).

13. The slope of the line is zero.
14. The slope of the line is undefined.
15. The slope of the line is positive, and its \( y \)-intercept is positive.
16. The slope of the line is positive, and its \( y \)-intercept is negative.
17. The slope of the line is negative, and its \( x \)-intercept is negative.
18. The slope of the line is negative, and its \( x \)-intercept is positive.

(a)  
(b)  
(c)  
(d)  
(e)  
(f)  
In Exercises 19 and 20, determine whether the lines through the pairs of points are parallel.

19. \(A(1, -2), B(-3, -10)\) and \(C(1, 5), D(-1, 1)\)

20. \(A(2, 3), B(2, -2)\) and \(C(-2, 4), D(-2, 6)\)

21. If the line passing through the points \((1, a)\) and \((4, -2)\) is parallel to the line passing through the points \((2, 8)\) and \((-7, a + 4)\), what is the value of \(a\)?

22. If the line passing through the points \((a, 1)\) and \((5, 8)\) is parallel to the line passing through the points \((4, 9)\) and \((a + 2, 1)\), what is the value of \(a\)?

In Exercises 23–26, find an equation of the line that passes through the point and has the indicated slope \(m\).

23. \((3, -4); m = 2\)

24. \((2, 4); m = -1\)

25. \((-3, 2); m = 0\)

26. \((1, 2); m = -\frac{1}{2}\)

In Exercises 27–30, find an equation of the line that passes through the given points.

27. \((2, 4)\) and \((3, 7)\)

28. \((3, 1)\) and \((3, 5)\)

29. \((1, 2)\) and \((-3, -2)\)

30. \((-1, -2)\) and \((3, -4)\)

In Exercises 31 and 32, determine whether the lines through the pairs of points are perpendicular.

31. \(A(-2, 5), B(4, 2)\) and \(C(-1, -2), D(3, 6)\)

32. \(A(2, 0), B(1, -2)\) and \(C(4, 2), D(-8, 4)\)

In Exercises 33–36, find an equation of the line that has slope \(m\) and \(y\)-intercept \(b\).

33. \(m = 3; b = 4\)

34. \(m = -1; b = -2\)

35. \(m = 0; b = 5\)

36. \(m = -\frac{1}{2}; b = \frac{3}{4}\)

In Exercises 37–42, write the equation in the slope-intercept form and then find the slope and \(y\)-intercept of the corresponding line.

37. \(x - 2y = 0\)

38. \(y - 8 = 0\)

39. \(2x - 3y - 9 = 0\)

40. \(3x - 4y + 8 = 0\)

41. \(2x + 4y = 14\)

42. \(5x + 8y - 24 = 0\)

43. Find an equation of the horizontal line that passes through \((-4, -3)\).

44. Find an equation of the vertical line that passes through \((0, 5)\).

45. Find an equation of the line that passes through the point \((-2, 2)\) and is parallel to the line \(3x - 4y - 8 = 0\).

46. Find an equation of the line that passes through the point \((-1, 3)\) and is parallel to the line passing through the points \((-2, -3)\) and \((2, 5)\).

47. Find an equation of the line that passes through the point \((2, 4)\) and is perpendicular to the line \(3x + 4y - 22 = 0\).

48. Find an equation of the line that passes through the point \((1, -2)\) and is perpendicular to the line passing through the points \((-2, -1)\) and \((4, 3)\).

49. Find an equation of the line that has slope \(-2\) and passes through the midpoint of the line segment joining the points \(P_1(-2, -4)\) and \(P_2(3, 6)\).


50. Find an equation of the line that passes through the midpoint of the line segment joining the points \(P_1(-1, -3)\) and \(P_2(3, 3)\) and the midpoint of the line segment joining the points \(P_3(-2, 3)\) and \(P_4(2, -3)\).


In Exercises 51–56, find an equation of the line that satisfies the given condition.

51. The line parallel to the \(x\)-axis and 4 units below it

52. The line passing through the origin and parallel to the line passing through the points \((2, 4)\) and \((4, 7)\)

53. The line passing through the point \((a, b)\) with slope equal to zero

54. The line passing through \((-3, 4)\) and parallel to the \(x\)-axis

55. The line passing through \((-5, -4)\) and parallel to the line passing through \((-3, 2)\) and \((6, 8)\)

56. The line passing through \((a, b)\) with undefined slope

57. Given that the point \(P(-3, 5)\) lies on the line \(kx + 3y + 9 = 0\), find \(k\).

58. Given that the point \(P(2, -3)\) lies on the line \(-2x + ky + 10 = 0\), find \(k\).

In Exercises 59–64, sketch the straight line defined by the linear equation by finding the \(x\)- and \(y\)-intercepts.

**Hint:** See Example 12.

59. \(3x - 2y + 6 = 0\)

60. \(2x - 5y + 10 = 0\)

61. \(x + 2y - 4 = 0\)

62. \(2x + 3y - 15 = 0\)

63. \(y + 5 = 0\)

64. \(-2x - 8y + 24 = 0\)
65. Show that an equation of a line through the points \((a, 0)\) and \((0, b)\) with \(a \neq 0\) and \(b \neq 0\) can be written in the form
\[
\frac{x}{a} + \frac{y}{b} = 1
\]
(Recall that the numbers \(a\) and \(b\) are the \(x\)- and \(y\)-intercepts, respectively, of the line. This form of an equation of a line is called the intercept form.)

In Exercises 66–69, use the results of Exercise 65 to find an equation of a line with the \(x\)- and \(y\)-intercepts.

66. \(x\)-intercept 3; \(y\)-intercept 4
67. \(x\)-intercept \(-1\); \(y\)-intercept \(-3\)
68. \(x\)-intercept \(-\frac{1}{2}\); \(y\)-intercept \(-\frac{3}{4}\)
69. \(x\)-intercept 4; \(y\)-intercept \(-\frac{1}{2}\)

In Exercises 70 and 71, determine whether the points lie on a straight line.

70. \(A(-1, 7), B(2, -2)\), and \(C(5, -9)\)
71. \(A(-2, 1), B(1, 7)\), and \(C(4, 13)\)

72. John claims that the following points lie on a line:
\((1.2, -9.04), (2.3, -5.96), (4.8, 1.04)\), and \((7.2, 7.76)\).
Prove or disprove his claim.

73. Alison claims that the following points lie on a line:
\((1.8, -6.44), (2.4, -5.72), (5.0, -2.72)\), and \((10.4, 3.88)\).
Prove or disprove her claim.

74. **Temperature Conversion** The relationship between the temperature in degrees Fahrenheit (°F) and the temperature in degrees Celsius (°C) is
\[
F = \frac{9}{5} C + 32
\]

(a) Sketch the line with the given equation.
(b) What is the slope of the line? What does it represent?
(c) What is the \(F\)-intercept of the line? What does it represent?

75. **Nuclear Plant Utilization** The United States is not building many nuclear plants, but the ones it has are running at nearly full capacity. The output (as a percentage of total capacity) of nuclear plants is described by the equation
\[
y = 1.9467t + 70.082
\]
where \(t\) is measured in years, with \(t = 0\) corresponding to the beginning of 1990.

(a) Sketch the line with the given equation.
(b) What are the slope and the \(y\)-intercept of the line found in part (a)?

(c) Give an interpretation of the slope and the \(y\)-intercept of the line found in part (a).

(d) If the utilization of nuclear power continued to grow at the same rate and the total capacity of nuclear plants in the United States remained constant, by what year were the plants generating at maximum capacity?

*Source: Nuclear Energy Institute.*

76. **Social Security Contributions** For wages less than the maximum taxable wage base, Social Security contributions (including those for Medicare) by employees are 7.65% of the employee’s wages.

(a) Find an equation that expresses the relationship between the wages earned \((x)\) and the Social Security taxes paid \((y)\) by an employee who earns less than the maximum taxable wage base.

(b) For each additional dollar that an employee earns, by how much is his or her Social Security contribution increased? (Assume that the employee’s wages are less than the maximum taxable wage base.)

(c) What Social Security contributions will an employee who earns $85,000 (which is less than the maximum taxable wage base) be required to make?

*Source: Social Security Administration.*

77. **College Admissions** Using data compiled by the Admissions Office at Faber University, college admissions officers estimate that 55% of the students who are offered admission to the freshman class at the university will actually enroll.

(a) Find an equation that expresses the relationship between the number of students who actually enroll \((y)\) and the number of students who are offered admission to the university \((x)\).

(b) If the desired freshman class size for the upcoming academic year is 1100 students, how many students should be admitted?

78. **Weight of Whales** The equation \(W = 3.51L - 192\), expressing the relationship between the length \(L\) (in feet) and the expected weight \(W\) (in British tons) of adult blue whales, was adopted in the late 1960s by the International Whaling Commission.

(a) What is the expected weight of an 80-ft blue whale?

(b) Sketch the straight line that represents the equation.

79. **The Narrowing Gender Gap** Since the founding of the Equal Employment Opportunity Commission and the passage of equal-pay laws, the gulf between men’s and women’s earnings has continued to close gradually. At the beginning of 2000 \((t = 0)\), women’s wages were 68% of men’s wages, and by the beginning of 2013 \((t = 13)\), women’s wages were 78% of men’s wages. If this gap between women’s and men’s wages continues to narrow *linearly*, then what percentage of men’s wages will women’s wages be at the beginning of 2020?

*Source: Journal of Economic Perspectives.*