1. **What is this course about**

The basic objects of Discrete Convex Geometry are integer solutions of systems of linear inequalities. These questions are almost as old as mathematics itself, and, perhaps more importantly, they naturally appear in a diverse array of mathematics-related disciplines, most notably in Operations Research and Economics.

Toric varieties are some particular examples of algebraic varieties that can be constructed from Discrete Convex Geometry objects. By virtue of their definition, they relate Discrete Convex Geometry and Algebraic Geometry, but they also have deep connections to other branches of Mathematics (Number Theory, Geometry of Numbers, Combinatorics, to name a few) as well as Mathematical Physics, Operations Research and Computer Science ([Bor4], [LZ], [E], [G], [KS], [S]).

Since Algebraic Geometry requires a background in Commutative Algebra, which most undergraduates lack, our approach to the subject will be elementary. Rather than proudly developing the general theory, we will be focusing on the series of examples of increasing level of difficulty. Perhaps the main goal of this course is to foster the development of the students’ geometrical intuition. We will also outline some of the many connections of this subject to other areas of human knowledge.

**Prerequisites.** For most projects the prerequisites include Linear Algebra, Introduction to Analysis, and Introduction to Algebra (at the level of Pitt courses Math 1180, Math 0413 and Math 0430). Depending on the course load in mathematics, these prerequisites are normally satisfied by mathematics majors after second or third year in college. Some experience with computer software (Maple or similar software, or a general programming language) is desired. The last project does not require any computer skills, but it requires some knowledge of Number Theory.

2. **Undergraduate research vision**

In my opinion, an ideal area for undergraduate research in pure mathematics, must have the following general features:
1) It must contain many problems that can be easily explained to an undergraduate;
2) These problems should be solvable without (or with limited use of) complicated mathematical machinery;
3) These problems must be unsolved, or not solved in a satisfactory manner;
4) The area must be connected to many active areas of mathematics and related disciplines;
5) The insight developed by working in this area can help the students in many other research endeavors.

Toric varieties satisfy all of the above. In fact, I can personally attest to this, as my first research, as an undergraduate, was on toric Fano varieties ([BB]). I have since worked in many areas of pure mathematics, but toric varieties still hold a special place in my heart, and the geometric intuition that they inspired has served me well. I hope to share this passion with the new generation of researchers.

3. Applications and broader impact

As mentioned above, Discrete Convex Geometry is the mathematical basis for a number of real-life problems in Operations Research, Economics, and other areas. However, no standard courses help the students to develop the geometric intuition for these problems. The geometry is, in general, a weak link for many students, since it is usually not adequately studied in high school and is almost never offered at the college level beyond the Linear Algebra. The hands-on experience with the Discrete Convex Geometry objects will boost the students’ intuition and confidence, that will serve them well in graduate studies in several mathematics-related disciplines.

4. Suggested projects

(1) Classification of 4-dimensional non-cyclic canonical toric singularities. This is unfinished business from my recent paper [BBBK]. No deep background is required, the result would be publishable.
(2) Toric surface singularities of minimal log discrepancy at least 1/2 (more advanced: at least 1/3). These are of interest for birational geometry ([Bor2], [R]).
(3) Nash resolution conjecture for Toric Varieties. A very intriguing relatively recent conjecture. The research will involve numerical experiments using computers ([ALPPT]).
(4) Accumulation conjecture for minimal log discrepancies of toric singularities. This is unfinished business from my 1997 paper [Bor1], that has received renewed attention recently.
(5) Effective theorem of Lawrence. This most challenging project is about an important and beautiful paper of Jim Lawrence [L], which
is, unfortunately, not effective. The goal is to use the ideas of Lagarias and Ziegler [LZ] to rewrite the proof, making explicit estimates. This project requires some background and interest in Number Theory.

5. References


