Review problems for Final Exam

1. Calculate $\int_0^1 \int_x^1 e^{x/y}$ by first reversing the order of integration.

2. Use polar coordinates to evaluate

$$\int_0^{\sqrt{2}} \int_y^{\sqrt{4-y^2}} \frac{1}{1+x^2+y^2} dxdy$$

3. Let $f(x,y,z) = x + 2y + z$ and R be the solid $x^2 + y^2 + z^2 \leq 4$, $\sqrt{x^2+y^2} \leq z$. Set up an integral to compute $\iiint_R f(x,y,z)dV$ (a) using rectangular coordinates, (b) using cylindrical coordinates, and (c) using spherical coordinates. DO NOT evaluate the integrals.

4. Use spherical coordinates to evaluate

$$\int_0^1 \int_0^{\sqrt{1-x^2}} \int_0^{\sqrt{1-x^2-y^2}} (x^2+y^2+z^2)^2 dzdydx$$

5. Consider the sphere $x^2 + (y-3)^2 + z^2 = 25$ and the cylinder $x^2 + y^2 = 4$. Set up (but do NOT evaluate) an integral which calculates the volume of the intersection of the inside of the sphere and the inside of the cylinder.

6. Use Green's Theorem to evaluate $\int_C (1 + tan(x))dx + (x^2 + e^y)dy$ where C is the positively oriented boundary of the region enclosed by the curves $y = \sqrt{x}$, $x = 1$, and $y = 0$.

7. Consider the vector field

$$\mathbf{F} = \frac{-y}{x^2+y^2}\mathbf{i} + \frac{x}{x^2+y^2}\mathbf{j}$$

Evaluate directly the line integral of $\mathbf{F}$ along the unit circle, once around in the counter-clockwise direction. Is $\mathbf{F}$ conservative? Compute the curl of $\mathbf{F}$. Why does your answer not contradict Green's Theorem?