Review problems for Exam 2

1. Use polar coordinates to find the exact value of the integral $\int_R y dA$ where R is the region in the first quadrant bounded by the circle $x^2 + y^2 = 1$ and the lines $y = x$ and $y = 0$.

2. By changing the order of integration, evaluate exactly the integral:

$\int_0^3 \int_{y^2}^9 y\cos(x^2)dxdy.$

3. Evaluate the following iterated integral:

$\int_0^\pi \int_0^1 \int_0^{\sqrt{1-y^2}} y \sin(x)\ dzdydx$

4. Find all critical points (x, y) of the function $f(x, y) = 3x^3 + y^2 - 9x + 4y$ and classify each as a local maximum, a local minimum, or a saddle point.

5. Use Lagrange multipliers to find the exact minimum value of $f(x, y, z) = 2x^2+y^2+3z^2$ subject to the constraint $2x - 3y - 4z = 49$.

6. Suppose that the temperature T (in degrees Celsius) at the point (x, y) is given by $T(x, y) = 10 + 3x^2 - 4y^2$.

(a) Find the direction $\mathbf{v}$ in which a bumblebee at the point $P(4, 3)$ should initially fly in order to get warmer in the shortest amount of time.

(b) Find the exact directional derivative of T at the point $P(4, 3)$ in the direction from P to $Q(5, 2)$.