

**Assignment 9 for Math 2301 Fall 2009**

**The due date for this assignment is Friday, November 6.**

1. (a). Let  $(X, \mu)$  be a measure space with  $\mu(X) < \infty$ . Let  $1 \leq p < q \leq \infty$ , show that

$$L^q(\mu) \subset L^p(\mu).$$

(Hint: Using Holder's inequality)

- (b). Construct a function  $f \in L^2(\mathbb{R})$  such that  $f \notin L^p(\mathbb{R})$  for any  $p \neq 2$ .
2. Let  $(X, \mu)$  be a measure space and  $1 \leq p < r < q \leq \infty$ . If  $f \in L^p(\mu) \cap L^q(\mu)$ , show that  $f \in L^r(\mu)$ .
3. Let  $(X, \mu)$  be a measure space with  $\mu(X) < \infty$  and  $f \in L^\infty(\mu)$ . Show that

$$\lim_{p \rightarrow \infty} \|f\|_p \rightarrow \|f\|_\infty.$$