

Assignment 12 for Math 2301 Fall 2009

The due date for this assignment is Friday, December 4.

1. Let $E \subset [0, 1]$ be the Cantor set created by repeatedly deleting the open middle thirds of a set of line segments. Let $I_1^n, \dots, I_{2^{n-1}}^n$ be the 2^{n-1} open intervals removed in the n -th step with the right order. Now define a function $f : [0, 1] \rightarrow [0, 1]$ such that
 - (i). $f(0) = 0$;
 - (ii). if $x \in I_i^n$ for some $1 \leq i \leq 2^{n-1}$, then $f(x) = \frac{2i-1}{2^n}$;
 - (iii). if $x \in C$ and $x \neq 0$, then

$$f(x) = \sup \{f(t) : t < x \text{ and } t \in [0, 1] \setminus C\}.$$

Show that f is continuous, monotone increasing and $f' = 0$ a.e..

2. Let $f, g \in BV[a, b]$ and $g \geq 1$ on $[a, b]$. Show that $\frac{f}{g} \in BV[a, b]$.