Assignment 1 for Math 2301 Fall 2009

The due date for this assignment is Wednesday, September 9.

1. **Let X be a nonempty set, and let $\mathcal{F}$ be a collection of subsets in X. Show that any element of the σ-algebra generated by $\mathcal{F}$ belongs to the σ-algebra generated by some countable subcollection of $\mathcal{F}$.

2. Let X be an uncountable set, and $\mathcal{F}$ is the collection of one-point subsets of X. Prove that the σ-algebra generated by $\mathcal{F}$ is

$$\mathcal{M} = \{E \subset X : E \text{ or } E^c \text{ is at most countable.}\}$$

3. Let $\{a_n\}_{n=1}^{\infty}$ be a sequence of nonnegative numbers. Define $\mu : 2^{\mathbb{N}} \to [0, \infty]$ by $\mu(\emptyset) = 0$ and for each nonempty set A of $\mathbb{N}$,

$$\mu(A) = \sum_{n=A} a_n.$$

Show that μ is a measure.

4. **Let μ be an outer measure on X. Assume that a subset A of X has the property that for each $\varepsilon > 0$, there exists a measurable set E such that $\mu(A \triangle E) < \varepsilon$. Show that A is also measurable. Here $A \triangle E$ is symmetric difference.

5. Let μ be an outer measure on X. Show that for every measurable set E and every subset $A \subset X$, we have

$$\mu(E) + \mu(A) = \mu(E \cup A) + \mu(E \cap A).$$