1. Find the area of the region bounded by the curves $y = 2x^2$ and $y = 6x - x^2$.

2. Find the volume of the solid obtained by rotating the region bounded by $\sqrt{\sin x}$, $y = 0$, $0 \leq x \leq \pi$, about the x-axis.

3. Evaluate the integral

$$\int_0^{\infty} x e^{-x^2} \, dx.$$

4. A bacteria culture grows with constant relative growth rate. At noon the culture initially contained $1,000$ cells. At 1pm, it contains $8,000$ cells. At what time will the population reach $32,000$ cells. Simplify your answer as much as possible.

5. Determine whether the sequence converges or diverges:

   (a)
   $$\{n^3 e^{-n}\}$$

   (b)
   $$\left\{ \frac{(-1)^n n}{2n+1} \right\}$$

   (c)
   $$\{\ln(2n-1) - \ln(n+1)\}$$

6. Find the sum of the series

   (a)
   $$\sum_{n=1}^{\infty} \frac{2^n}{3^{n-1}}$$

   (b)
   $$\sum_{n=1}^{\infty} \left(2^{1/n} - 2^{1/(n+1)} \right)$$

   (c)
   $$\sum_{n=1}^{\infty} \frac{(-1)^n}{n!}$$

7. Determine whether the series converges or diverges

   (a)
   $$\sum_{n=2}^{\infty} \frac{1}{n\sqrt{\ln n}}$$

(b)

$$\sum_{n=0}^{\infty} \frac{5^n}{n!}$$

(c)

$$\sum_{n=1}^{\infty} \frac{(-1)^n}{n^2+1}$$

(d)

$$\sum_{n=1}^{\infty} \frac{1}{2^{1/n}}$$

8. Represent the integral

$$\int \sin\left(x^2\right) dx$$

as a Maclaurin series.

9. Find the Taylor polynomial of degree 3 for the function $f(x) = x^5$ at $a = -1$.

10. Solve the differential equation

$$y'' - 6y' + 9 = 0.$$

11. Solve the differential equation

$$xy' - y = x^3 \sin x.$$

12. Find a unit vector perpendicular to the plane containing the points $A(0,1,3)$, $B(2,0,3)$ and $C(2,1,4)$.