

Sample Problems for the Second Midterm for Math 0230
November 21, 2011

1. Find the sum of the convergent geometric series

$$3 - 2 + \frac{4}{3} - \frac{8}{9} + \frac{16}{27} - \dots$$

2. Determine whether the series is convergent

(a)

$$\sum_{n=0}^{\infty} \frac{2^{3n+10}}{3^{2n+1}};$$

(b)

$$\sum_{n=1}^{\infty} \frac{n^2 + 2n + 2011}{2n^4 + n - 2};$$

(c)

$$\sum_{n=1}^{\infty} \sin \frac{1}{n};$$

(d)

$$\sum_{n=1}^{\infty} (-1)^n \frac{1}{1 + \sqrt{n}};$$

(e)

$$\sum_{n=0}^{\infty} \frac{2^{3n} 5^n}{n!},$$

(f)

$$\sum_{n=5}^{\infty} \frac{1}{n \ln^2 n}.$$

3. Determine the radius of convergence and interval of convergence for

(a)

$$\sum_{n=0}^{\infty} \frac{(2x - 3)^{3n}}{3^{2n+1}};$$

(b)

$$\sum_{n=1}^{\infty} \frac{x^n}{2n + 1};$$

(c)

$$\sum_{n=1}^{\infty} (-1)^n n^2 2^n (x+1)^n.$$

4. Determine the Taylor series about $x = 0$ and find its radius of convergence.

(a)

$$\frac{1}{2x+3};$$

(b)

$$\sqrt{1+2x};$$

(c)

$$xe^{-x^2}.$$

5. Find the Taylor polynomial T_7 for

$$f(x) = e^{x^3} \sin x.$$

6. Consider the parametric curve

$$x = \frac{1}{3}t^3 - t, y = t^2,$$

Find equations of tangent lines at $(0, 3)$.

7. Find the slope of the tangent line for the curve $r = 2 - \sin \theta$ when $\theta = \frac{\pi}{6}$.

8. Find the length of the curve $r = 3 \sin \theta + 4 \cos \theta$, $0 \leq \theta \leq \frac{\pi}{2}$.

9. Find the length of the curve $x = \theta - \cos \theta$, $y = 1 - \sin \theta$, $0 \leq \theta \leq \pi$.

10. Given point $P(1, -1)$, find its polar coordinate.

11. Find a Cartesian equation for the polar curve $r = 2 \sin \theta$, then sketch the curve. Find the area of the region that lies inside $r = 2 \sin \theta$ and outside of $r = 1$.