

Sample Problems for First Midterm for Math 0230
November 7, 2007

1. Find the sum of the geometric series

$$3 - 2 + \frac{4}{3} - \frac{8}{9} + \frac{16}{27} - \dots$$

2. Determine whether the series is convergent.

(a)

$$\sum_{n=0}^{\infty} \frac{2^{3n+10}}{3^{2n+1}};$$

(b)

$$\sum_{n=1}^{\infty} \frac{n^2 + 3n + 1000}{2n^4 + n - 2};$$

(c)

$$\sum_{n=3}^{\infty} \frac{(-1)^n \ln n}{n + 1};$$

(d)

$$\sum_{n=3}^{\infty} (-1)^n \sin(\sqrt{n});$$

(e)

$$\sum_{n=0}^{\infty} \frac{99^{7n+3}}{n!};$$

(f)

$$\sum_{n=0}^{\infty} \frac{2^{3n+10} n^5}{3^{2n+1}};$$

3. Find all values of p , so that

$$\sum_{n=3}^{\infty} \frac{1}{n (\ln n)^p}$$

is convergent. Show your reasoning.

4. Find all solutions to the second order nonhomogeneous differential equation

$$y'' + 2y' - 3y = \sin 2x.$$

5. Solve the initial value problem to the first order differential equation

$$y' = x + y, \quad y(0) = 5.$$

6. Determine the radius of convergence and interval of convergence for

(a)

$$\sum_{n=0}^{\infty} \frac{(2x - 3)^{3n}}{3^{2n+1}};$$

(b)

$$\sum_{n=0}^{\infty} \frac{x^n}{2n + 1};$$

(c)

$$\sum_{n=0}^{\infty} (-1)^n n^2 2^n (x + 1)^n.$$

7. Determine the Taylor series about $x = 0$ for

(a)

$$\frac{1}{2x + 3};$$

(b)

$$\sqrt{1 - 2x};$$

(c)

$$x^2 \tan^{-1} x.$$

8. Let $f(x) = e^x \sin x^7$. Find $f^{(10)}(0)$.