

Sample Problems for First Midterm for Math 0230  
October 1, 2008

1. Evaluate the following integrals

(a)

$$\int_1^{\infty} \frac{1}{1+x^2} dx;$$

(b)

$$\int_0^{\infty} x e^{-x} dx;$$

(c)

$$\int_0^1 \sqrt{1-x^2} dx;$$

(d)

$$\int_{\frac{1}{2}}^1 \frac{1}{\sqrt{1-x^2}} dx;$$

(e)

$$\int \sin^3 x \cos^2 x dx;$$

(f)

$$\int x \cos x dx;$$

(g)

$$\int \frac{x}{(x-1)(x-2)} dx.$$

2. Find the average value of  $\frac{3}{1+4x^2}$  on  $0 \leq x \leq \frac{1}{2}$ .

3. Find the volume of the solid obtained by rotating the triangle region bounded by  $y = \sqrt{x}$  and  $y = x$  about  $y$  axis.

4. Find the volume of the solid obtained by rotating the triangle region bounded by  $y = x, x = 0$  and  $y = 3 - 2x$  about  $y$  axis.

5. Find the solution of

$$y' = xy^2 + x$$

satisfying initial condition  $y(0) = 1$ .

6. Find all solutions of

$$\frac{dy}{dt} = y^2 + y.$$

7. Use Euler's method with a step size of  $\frac{1}{4}$  to approximate  $y\left(\frac{1}{2}\right)$  if  $y' = xy+1$  and  $y(0) = 0$ .
8. Calculate the length of the curve described by the parametric equation

$$x = t^2, y = 2 \int_0^t \sqrt{s - s^2} ds, 0 \leq t \leq 1.$$

9. Find the work done in pumping the water out of a the top of spherical tank of radius  $3m$  that is full of water.
10. A tank contains  $10kg$  of salt dissolved in  $1000L$  of water. Brine that contains  $0.02kg$  of salt per liter of water enters the tank at a rate of  $5L/\text{min}$ . The solution is kept thoroughly mixed and drains from the tank at the same rate. How much salt remains in the tank after 40 minutes?
11. When a cold drink is taken from a refrigerator, its temperature is  $5^\circ C$ . After 20 minutes in a  $20^\circ C$  room its temperature has increased to  $10^\circ C$ . When will its temperature be  $15^\circ C$ ?
12. Find the hydrostatic force on a flat triangle plate with sides  $3m$ ,  $4m$  and  $5m$  which is submerged in  $10m$  of water. Here the plate is placed vertically so that the top of the plate is the  $3m$  side, which is horizontal and  $10m$  away from the water surface.