

Sample Problems for First Midterm for Math 0230  
October, 2011

1. Evaluate the following integrals

(a) 
$$\int_1^{\infty} \frac{1}{1+x^2} dx;$$

(b) 
$$\int_0^{\infty} x e^{-x} dx;$$

(c) 
$$\int_0^1 \sqrt{1-x^2} dx;$$

(d) 
$$\int_{\frac{1}{2}}^1 \frac{1}{\sqrt{1-x^2}} dx;$$

(e) 
$$\int \sin^3 x \cos^2 x dx;$$

(f) 
$$\int x \cos x dx;$$

(g) 
$$\int \frac{x}{(x-1)(x-2)} dx.$$

2. Find the volume of the solid obtained by rotating the triangle region bounded by  $y = \sqrt{x}$  and  $y = x$  about  $y$  axis.

3. Find the volume of the solid obtained by rotating the triangle region bounded by  $y = x$ ,  $x = 0$  and  $y = 3 - 2x$  about  $y$  axis.

4. Calculate the length of the curve described by the parametric equation

$$x = t^2, y = 2 \int_0^t \sqrt{s - s^2} ds, 0 \leq t \leq 1.$$

5. Find the work done in pumping the water out of a the top of spherical tank of radius  $3m$  that is full of water.

6. Find the hydrostatic force on a flat triangle plate with sides  $3m$ ,  $4m$  and  $5m$  which is submerged in  $10m$  of water. Here the plate is placed vertically so that the top of the plate is the  $3m$  side, which is horizontal and  $10m$  away from the water surface.
7. Find the area of the triangle  $ABC$  for  $A(1, 2, 3)$ ,  $B(3, 4, 2)$  and  $C(5, 3, 4)$ .
8. Find the volume of the parallelepiped formed by the vectors

$$\vec{a} = \langle 1, 2, 3 \rangle, \vec{b} = \langle 2, -3, 1 \rangle \text{ and } \vec{c} = \langle 3, 1, 2 \rangle.$$

9. Find the angle between vector  $\overrightarrow{AB}$  and vector  $\overrightarrow{AC}$  for  $A(1, 2, 3)$ ,  $B(2, 3, 1)$  and  $C(3, 1, 2)$ .
10. Determine the symmetric equation of the line passing through  $A(2, 3, 1)$  and  $B(3, 1, 2)$ .
11. Find the equation of the plane which contains the points  $A(1, 2, 3)$ ,  $B(2, 3, 1)$  and  $C(3, 1, 2)$ .
12. Find the equation of the plane which is parallel to vectors  $\vec{u} = \langle 1, -2, 3 \rangle$  and  $\vec{v} = \langle 3, -2, 1 \rangle$  and contains point  $P(3, 1, 2)$ .
13. Find the distance between two parallel planes  $2x + y + 2z = 3$  and  $4x + 2y + 4z = 1$ .