1. Fine Art and Craft Materials had sales of $60,000 in its first year of operation. If sales increased by 4% per year thereafter, find the company’s sales in the seventh year and its total sales over the first ten years of operation.

Solution: It is a geometric progression problem with $a_1 = 60,000$ and $r = 1 + 0.04 = 1.04$. We have to find $a_7$ and $S_{10}$.

$$a_7 = 60,000 \cdot 1.04^{7-1} = 60,000 \cdot 1.04^6 = 75,919.14$$

$$S_{10} = 60,000 \cdot \frac{1 - 1.04^{10}}{1 - 1.04} = 720,366.43$$

Answer: sales in the seventh year are $75,919.14$

total sales over the first ten years are $720,366.43$

2. Consumer Surveys. In a survey of 180 customers conducted in a shopping mall, 100 customers indicated that they buy Brand A of a certain product, 125 customers indicated that they buy Brand A of a certain product, and 64 indicated that they buy both brands. How many customers participating in the survey buy

(a) At least one of these brands?
(b) Only brand A?
(c) Exactly one of these brands?
(d) Neither of these brands?

Solution: (a) Denote by $U$ the universal set, by $A$ the set of customers indicated that they buy Brand A, and by $B$ the set of customers indicated that they buy Brand B. Then $n(U) = 180$, $n(A) = 100$, $n(B) = 125$, $n(A \cap B) = 64$.

(a) $n(A \cup B) = n(A) + n(B) - n(A \cap B) = 100 + 125 - 64 = 161$

(b) $n(A) - n(A \cap B) = 100 - 64 = 36$

(c) $(n(A) - n(A \cap B)) + (n(B) - n(A \cap B)) = 36 + 61 = 97$

(d) $n(U) - n(A \cup B) = 180 - 161 = 19$

3. Senate Committees. In how many ways can a subcommittee of six can be chosen from a Senate committee of seven Democrats and six Republicans if the subcommittee must contain of three Democrats and three Republicans?

Solution: There are $C(7, 3)$ combinations to chose three Democrats out of seven and $C(6, 3)$
combinations to chose three Republicans out of six. The total number of combinations is
\[ C(7, 3) \cdot C(6, 3) = \frac{7!}{3!4!} \cdot \frac{6!}{3!3!} = \frac{5 \cdot 6 \cdot 7 \cdot 4 \cdot 5 \cdot 6}{2 \cdot 3 \cdot 2 \cdot 3} = 700 \]

4. Let \( E \) and \( F \) be two events of an experiment with sample space \( S \). Suppose \( P(E) = 0.6 \), \( P(F) = 0.4 \), and \( P(E \cap F) = 0.3 \). Compute:
   (a) \( P(E \cup F) \)
   (b) \( P(E^c) \)
   (c) \( P(E^c \cap F) \)

Solution: The events \( E \) and \( F \) are not mutually exclusive because \( P(E \cap F) \neq 0 \).
   (a) \( P(E \cup F) = P(E) + P(F) - P(E \cap F) = 0.6 + 0.4 - 0.3 = 0.7 \)
   (b) \( P(E^c) = 1 - P(E) = 1 - 0.6 = 0.4 \)
   (c) \( P(E^c \cap F) = P(F) - P(E \cap F) = 0.4 - 0.3 = 0.1 \)

5. A customer at CL Fruit Stand picks a sample of 4 oranges at random from a crate containing 50 oranges, of which 6 are rotten. What is the probability that the sample contains one or more rotten oranges? You can write the answer in a decimal form with four correct decimal places.

Solution: The sample space \( S \) is a set of all quadruples of four oranges. The event \( E \) is the set of all such quadruples that contains one or more rotten oranges. Then the event \( E^c \) is the set of all quadruples that contains no rotten oranges which means that it contains only good oranges.

\[ n(S) = C(50, 4). \]

The number of good oranges is \( 50 - 6 = 44 \). Then \( n(E^c) = C(44, 4) \).

\[ P(E) = 1 - P(E^c) = 1 - \frac{n(E^c)}{n(S)} = 1 - \frac{C(44, 4)}{C(50, 4)} = 1 - \frac{44 \cdot 43 \cdot 42 \cdot 41}{50 \cdot 49 \cdot 48 \cdot 47} \]
\[ = 1 - 0.58945 = 0.41055 = 0.4106. \]

6. A pair of fair dice is rolled. What is the probability that one of the numbers landing uppermost is a 5 if it is known, that the sum of the numbers landing uppermost is 8?

Solution: It is a conditional probability. Let \( A \) denote the event that one of the numbers landing uppermost is a 5 and \( B \) denote the event that the sum of the numbers landing uppermost is 8. Then

\[ A = \{(1, 5), (2, 5), (3, 5), (4, 5), (5, 5), (6, 5), (5, 1), (5, 2), (5, 3), (5, 4), (5, 6)\}, \]
\[ B = \{(2, 6), (3, 5), (4, 4), (5, 3), (6, 2)\}, \quad A \cap B = \{(3, 5), (5, 3)\}. \]

\[ P(A \cap B) = \frac{2}{36}, \quad P(B) = \frac{5}{36} \]

Then \[ P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{2}{5} = 0.4 \]

7. Quality Control The panels for LCD HDTV are manufactured in two locations: in Plants I and II. Plant I supplies 60% and Plant II supplies 40% of the panels. 1% of the panels produced by the Plant I and 3.5% of the panels produced by the Plant II are defective. If a panel is selected at random and the panel is found to be defective, what is the probability that the panel was manufactured in Plant II?

\textit{Solution:} Here we use Bayes’ formula. Let \( A \) denote the event that the panel was manufactured in Plant I, \( A^c \) denote the event that the panel was manufactured in Plant II and \( B \) denote the event that the panel is defective. Then

\[ P(A^c|B) = \frac{P(A^c) \cdot P(B|A^c)}{P(A) \cdot P(B|A) + P(A^c) \cdot P(B|A^c)} \]

\[ P(A) = 0.6, \quad P(B|A) = 0.01, \quad P(A^c) = 0.4, \quad P(B|A^c) = 0.035 \]

\[ P(A^c|B) = \frac{(0.4)(0.035)}{(0.6)(0.01) + (0.4)(0.035)} = \frac{0.014}{0.020} = \frac{14}{20} = 0.7 \]

8. The probability distribution of a random variable \( X \) is shown in the following table:

<table>
<thead>
<tr>
<th>( x )</th>
<th>3</th>
<th>5</th>
<th>6</th>
<th>8</th>
<th>12</th>
<th>15</th>
</tr>
</thead>
<tbody>
<tr>
<td>( P(X = x) )</td>
<td>.12</td>
<td>.22</td>
<td>.14</td>
<td>.19</td>
<td>.13</td>
<td>.2</td>
</tr>
</tbody>
</table>

Find: (a) \( P(x \leq 3) \) \quad (b) \( P(x \leq 6) \) \quad (c) \( P(x = 6) \) \quad (d) \( P(6 < x \leq 12) \) \quad (e) \( P(x > 8) \).

\textit{Solution:} (a) \( P(x \leq 3) = P(x = 3) = 0.12 \)

(b) \( P(x \leq 6) = P(x = 3) + P(x = 5) + P(x = 6) = 0.12 + 0.22 + 0.14 = 0.48 \)

(c) \( P(x = 6) = 0.14 \)

(d) \( P(6 < x \leq 12) = P(x = 8) + P(x = 12) = 0.19 + 0.13 = 0.32 \)

(e) \( P(x > 8) = P(x = 12) + P(x = 15) = 0.13 + 0.2 = 0.33 \)

\textit{bonus problem} Two cards are drawn in succession without replacement from a standard deck of 52 cards. What is the probability that the first card is an ace given that the second card is
a queen?

**Solution:** Here we use Bayes’ formula. Let $A$ denote the event that a drawn card is an ace, $Q$ denote the event that a drawn card is a queen, and $B$ denote the event that a drawn card is neither an ace nor a queen. The stage one represents the first drawn card and the stage two represents the second drawn card.

\[
P(A|Q) = \frac{P(A) \cdot P(Q|A)}{P(A) \cdot P(Q|A) + P(Q) \cdot P(Q|Q) + P(B) \cdot P(Q|B)}
\]

\[
P(A) = \frac{4}{52}, \quad P(Q|A) = \frac{4}{51}, \quad P(Q) = \frac{4}{51}, \quad P(Q|Q) = \frac{3}{51}, \quad P(B) = \frac{44}{52}, \quad P(Q|B) = \frac{4}{51}
\]

\[
P(B|D) = \frac{\frac{4}{52} \cdot \frac{4}{51} + \frac{4}{52} \cdot \frac{3}{51} + \frac{44}{52} \cdot \frac{4}{51}}{\frac{4}{52} + \frac{3}{51} + \frac{44}{52}} = \frac{4}{51}
\]
bonus problem There are seven cards: Ace, King, Queen and Jack of Diamonds and Ace, Queen and Jack of Hearts. Two cards are drawn in succession without replacement. What is the probability that the first card is an ace given that the second card is a queen?