MATH 4000 - Discrete Mathematical Structures

FINAL EXAMINATION

FALL 2011

NAME (Print): ________________________________

Student ID: ________________________________

NAME (Signature): __________________________

INSTRUCTIONS:

1. NO BOOKS, HEADPHONES, OR COMPUTERS MAY BE USED.
   You can use a calculator and a letter format formula sheet.

2. Before you begin, Enter your Name and Student ID Number in the space above.

3. Show ALL of your Work on the Exam itself. Write your answers in the corresponding boxes, if provided.

1. _____ (9)  2. _____ (8)  3. _____ (7)  4. _____ (8)  5. _____ (10)  6. _____ (8)

7. _____ (8)  8. _____ (9)  9. _____ (9)  10. _____ (8)  11. _____ (10)  12. _____ (6)

TOTAL: ________________ (100)
AutoTime, a manufacturer of 24-hr variable timers, has a monthly fixed cost of $39,000 and a production cost of $9 for each timer manufactured. The timers sell for $13 each.

(a) (2 points) Find the cost function.

Cost function:

(b) (2 points) Find the revenue function.

Revenue function:
(c) (2 points) Find the profit function.

Profit function:

(d) (3 points) Compute the profit or loss corresponding to production level of 5000, 8000, and 15,000 timers, respectively.

\[ P(5000) = \]

\[ P(8000) = \]

\[ P(15,000) = \]
2. (8 points) Diet Planning. A dietitian wishes to plan a meal around three foods. The percent of a daily requirements of proteins, carbohydrates, and iron in each ounce of the three foods is summarized in the following table:

<table>
<thead>
<tr>
<th></th>
<th>Food I</th>
<th>Food II</th>
<th>Food III</th>
</tr>
</thead>
<tbody>
<tr>
<td>Protein (%)</td>
<td>8</td>
<td>4</td>
<td>12</td>
</tr>
<tr>
<td>Carbohydrates (%)</td>
<td>10</td>
<td>6</td>
<td>6</td>
</tr>
<tr>
<td>Iron (%)</td>
<td>6</td>
<td>8</td>
<td>14</td>
</tr>
</tbody>
</table>

Determine how many ounces of each food the dietitian should include in the meal to meet exactly the daily requirement of proteins, carbohydrates, and iron (100% of each).
3. (This problem has two parts (a) and (b))

(a) (5 points) Construct a truth table for the compound proposition

\[(p \land \sim q) \rightarrow (p \lor \sim q)\]

(b) (2 points) Is the statement a tautology, a contradiction, or neither?

Answer: □
4. (8 points) Show that the argument is valid by writing it in the symbolic form and constructing a truth table:

- You return the book on time, or you will have to pay a fine.
- You do not return the book on time.

Therefore, you will have to pay a fine.

Circle your answer: valid invalid
5. (10 points) Financing a College Education. The parents of a child wish to establish a trust fund for her college education. They estimate that they will need $90,000 in 12 years. How much should they set aside in the trust now if they can invest the money at 7.5% per year compounded semiannually?

Answer: 


6. Consumer survey. (This problem has four parts (a), (b), (c), and (d))

In a survey of 160 consumers conducting in a shopping mall, 95 consumers indicated that they buy brand A of a certain product, 77 buy brand B, and 34 buy both brands. How many consumers participating in the survey buy

(a) (2 points) At least one of these brands?

Answer: 

Continued on the next page
(b) (2 points) Exactly one of these brands?

Answer: 

(c) (2 points) Only brand B?

Answer: 

(d) (2 points) None of these brands?

Answer: 
7. (8 points) Car Pools. A company car that has a seating capacity of seven is to be used by seven employees who have formed a car pool. If only five of these employees can drive, how many possible seating arrangements are there for the group? (Assume that all the employees are different persons).

Answer: 11
8. (This problem has three parts (a), (b), and (c))
Let \( S = \{s_1, s_2, s_3, s_4, s_5\} \), be the sample space associated with an experiment having the probability distribution shown in the table

<table>
<thead>
<tr>
<th>Outcome</th>
<th>( s_1 )</th>
<th>( s_2 )</th>
<th>( s_3 )</th>
<th>( s_4 )</th>
<th>( s_5 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Probability</td>
<td>1/3</td>
<td>1/10</td>
<td>1/5</td>
<td>1/6</td>
<td>1/5</td>
</tr>
</tbody>
</table>

If \( E = \{s_1, s_2, s_4\} \), \( F = \{s_2, s_3\} \), find

(a) (3 points) \( P(E \cap F) \)

Answer: 

Continued on the next page
(b) (3 points) $P(E \cup F^c)$

Answer: 

(c) (3 points) $P(E^c \cap F^c)$

Answer: 
9. Investment analysis. (This problem has three parts (a), (b), and (c))
Diane Walters is considering investing $16,000 in two mutual funds. The anticipated returns from price appreciation and dividends are distributed by the following probability distributions:

<table>
<thead>
<tr>
<th>Mutual Fund A:</th>
<th>Mutual Fund B:</th>
</tr>
</thead>
<tbody>
<tr>
<td>Returns</td>
<td>Returns</td>
</tr>
<tr>
<td>Probability</td>
<td>Probability</td>
</tr>
<tr>
<td>−$400</td>
<td>−$900</td>
</tr>
<tr>
<td>.3</td>
<td>.40</td>
</tr>
<tr>
<td>$200</td>
<td>$600</td>
</tr>
<tr>
<td>.4</td>
<td>.45</td>
</tr>
<tr>
<td>$800</td>
<td>$1400</td>
</tr>
<tr>
<td>.3</td>
<td>.15</td>
</tr>
</tbody>
</table>

(a) (5 points) Compute the mean value and variance associated with the returns for each mutual fund.
(b) (2 points) Which investment would provide Diane with the higher expected return (the greater mean)?

Circle your answer: Fund A  Fund B

(c) (2 points) In which investment would the element of risk be less (that is, which probability distribution has the smaller variance)?

Circle your answer: Fund A  Fund B
10. (8 points) Grade Distribution. The scores on an examination are normally distributed with a mean of 76 and a standard deviation of 14. If the instructor assigns A’s to 20% and B’s to 25% of the class what are the cutoff points for the grade B?

Answer:
11. Commuter Trends. (This problem has three parts (a), (b), and (c))
Within a large metropolitan area, 25% of the commuters currently use the public transportation system, whereas the remaining 75% commute via automobile. The city has recently revitalized and expanded its public transportation system. It is expected that 1 year from now 35% of those who are commuting to work via automobile will switch to public transportation and 65% will continue to commute via automobile. At the same time, it is expected that 15% of those now using public transportation will commute via automobile, and 85% will continue to use public transportation.

(a) (3 points) Draw a tree diagram of the process. Find the transition matrix and the initial distribution vector for the Markov chain that describes the change in the mode of transportation used by these commuters.

Draw your tree diagram here:
(b) (3 points) What percentage of the commuters are expected to use public transportation 1 year from now?

Answer: 

Continued on the next page
(c) (4 points) In the long run, what percentage of the commuters will be using public transportation?

Answer: 19
12. (6 points) Is the matrix

\[ A = \begin{bmatrix} 1 & .3 \\ 0 & .7 \end{bmatrix} \]

regular?

Circle your answer:  regular  not regular

Explain why it is regular or why it is not.