**Problem:** Evaluate \( I = \iint_S \text{curl} \vec{F} \cdot d\vec{S} \), where \( \vec{F} = xyz \vec{i} + x \vec{j} + e^{xy} \cos z \vec{k} \), and \( S \) is hemisphere \( x^2 + y^2 + z^2 = 1, \; z \geq 0 \) oriented upward.

**Solution:** By Stoke’s theorem \( I = \iint_S \text{curl} \vec{F} \cdot d\vec{S} = \oint_C \vec{F} \cdot d\vec{r} \), where \( C \) is the circle \( x^2 + y^2 = 1 \) oriented counterclockwise. Its parametrization is \( \vec{r}(t) = \langle \cos t, \sin t, 0 \rangle \). Then \( \vec{r}'(t) = \langle -\sin t, \cos t, 0 \rangle \), \( \vec{F}(\vec{r}(t)) = \langle 0, \cos t, e^{\cos t \sin t} \rangle \), \( \vec{F}(\vec{r}(t)) \cdot \vec{r}'(t) = \cos^2 t \).

\[
I = \int_0^{2\pi} \cos^2 t \, dt = \int_0^{2\pi} \frac{1}{2} (1 + \cos 2t) \, dt = \left[ \frac{1}{2} t + \frac{1}{4} \sin 2t \right]_0^{2\pi} = \pi
\]

**Problem:** Use Stoke’s theorem to evaluate \( I = \oint_C \vec{F} \cdot d\vec{r} \), where \( \vec{F} = x \vec{i} + y \vec{j} + (x^2 + y^2) \vec{k} \), and \( C \) is the boundary of the part of a paraboloid \( z = 1 - x^2 - y^2 \) in the first octant oriented counterclockwise as viewed from above.

**Solution:** By Stoke’s theorem \( I = \oint_C \vec{F} \cdot d\vec{r} = \iint_S \text{curl} \vec{F} \cdot d\vec{S} \)

where \( \text{curl} \vec{F} = 2y \vec{i} - 2x \vec{j} + 0 \vec{k} \), \( S: \; z = 1 - x^2 - y^2 = g(x,y) \), \( \vec{n} = \frac{\langle 2x, 2y, 1 \rangle}{\sqrt{4x^2 + 4y^2 + 1}} \).

Then \( \text{curl} \vec{F} \cdot \vec{n} = 0 \) and \( I = \iint_S 0 \, dS = 0 \)

**Problem:** Use the Divergence theorem to evaluate \( I = \iiint_B \text{div} \vec{F} \, dV \), where \( \vec{F} = x^4 z \vec{i} + 2x^3 yz \vec{j} + x^3 z^2 \vec{k} \) and \( S \) is the boundary surface of the box \( B \) with vertices \((0,0,0),(1,0,0),(0,2,0),(0,0,3),(1,2,0),(1,0,3),(0,2,3)\), and \((1,2,3)\) with outwards pointing normal vector.

**Solution:** By the Divergence theorem \( I = \iiint_B \text{div} \vec{F} \, dV = \iiint_S \vec{F} \cdot d\vec{S} \)

where \( \text{div} \vec{F} = 4x^3 z + 2x^3 z + 2x^3 z = 8x^3 z \), \( B = \{(x,y,z) \mid 0 \leq x \leq 1, 0 \leq y \leq 2, 0 \leq z \leq 3\} \).
Hence, 
\[ I = \int_0^1 \int_0^2 \int_0^3 8x^3z \, dz \, dy \, dx = 8 \int_0^1 x^3 \, dx \int_0^2 dy \int_0^3 z \, dz = 8 \cdot \frac{1}{4} \cdot 2 \cdot \frac{9}{2} = 18 \]

**Problem:** Use the Divergence theorem to evaluate \( I = \iiint_S \bar{F} \cdot d\bar{S} \), where 
\[ \bar{F} = y^2 z \bar{i} + y^3 \bar{j} + xz \bar{k} \] and \( S \) is the boundary surface of the box \( B \) defined by \(-1 \leq x \leq 1,\ 0 \leq y \leq 1,\ \text{and} \ 0 \leq z \leq 2 \) with outwards pointing normal vector.

**Solution:** By the Divergence theorem 
\[ I = \iiint_S \bar{F} \cdot d\bar{S} = \iiint_B \text{div} \bar{F} \, dV \]

where \( \text{div} \bar{F} = 0 + 3y^2 + x = x + 3y^2 \). \( B = \{(x, y, z) \mid -1 \leq x \leq 1, 0 \leq y \leq 1, 0 \leq z \leq 2\} \). Hence, 
\[ I = \int_{-1}^1 \int_0^2 \int_0^3 (x + 3y^2) \, dz \, dy \, dx = 2 \int_{-1}^1 \int_0^2 (x + 3y^2) \, dy \, dx = 2 \int_{-1}^1 (x + 1) \, dx = 4 \]

**Bonus problem:** Evaluate the flux of the vector field \( \bar{F} = \langle x^2 y^2, -xy^3, xy^2 z - e^{x+y} \rangle \) across the surface \( S \) of the solid \( E \) bounded by the hyperboloid \( x^2 + y^2 - z^2 = -1 \) and the paraboloid \( z = 4 - x^2 - y^2 \), when \( z \geq 0 \).
[ :-) Have a wonderful day! ]

**Solution:** The flux is the surface integral \( \iint_S \bar{F} \cdot d\bar{S} \). 
\[ \text{div} \bar{F} = 2xy^2 - 3xy^2 + xy^2 = 0 \]

By the Divergence theorem 
\[ \iint_S \bar{F} \cdot d\bar{S} = \iiint_E 0 \, dV = 0. \]